University of Toronto
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Working Paper 779

The Micro and Macro Productivity of Nations

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July 18, 2024

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Abstract

We examine aggregate productivity differences across nations using cross-country firm-level data and a quantitative model of production heterogeneity with distortions featuring operation decisions (selection) and productivity-enhancing investments (technology). Empirically, less developed countries feature higher distortions and larger dispersion in firm-level productivity, mostly resulting from the higher prevalence of unproductive firms. Quantitatively, measured cross-country differences in the elasticity of distortions with respect to firm productivity generate the bulk of empirical patterns and over two-thirds of cross-country labor productivity differences. Both selection and technology channels are important. Variation in static misallocation also plays an important role, albeit smaller.

Keywords: Firms, productivity, size, distortions, misallocation, selection, technology.
JEL classification: O11, O14, O4.

*We are grateful for useful comments to Paco Buera, Marcela Eslava, Hugo Hopenhayn, Venky Venkateswaran, and audiences at Toronto, LACEA-LAMES 2023 in Bogota, Hong Kong Macroeconomics Workshop, UNC Chapel Hill, Econometric Society NASM 2024 in Nashville, and ThReD 2024 in Namur. All remaining errors are our own. Restuccia gratefully acknowledges the support from the Canada Research Chairs program and the Bank of Canada Fellowship program. The views expressed herein are those of the authors and should not be attributed to the Bank of Canada or its Governing Council, nor the IMF, its Executive Board, or its management. Contact: stephen.b.ayerst@gmail.com, ducm.nguyen@mail.utoronto.ca, and diego.restuccia@utoronto.ca.

1 Introduction

There are large disparities in aggregate productivity across countries which are at the core of international differences in GDP per capita (Klenow and Rodriguez-Clare, 1997; Prescott, 1998; Hall and Jones, 1999). Cross-country differences in aggregate productivity are linked to distortions in the allocation of resources across firms within sectors (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Bartelsman et al., 2013). Whereas the misallocation literature emphasizes the aggregate productivity gains from factor reallocation across a given set of producers, producer-level data also reveal substantial differences in the productivity distribution across countries (Hsieh and Klenow, 2009; Gal, 2013; Andrews et al., 2015). In this paper, we follow Restuccia and Rogerson (2017) in linking observed firm-level TFP distributions to policies and institutions that misallocate resources across firms. This approach is motivated by empirical evidence from policy reforms that find substantial improvements in selection and technology upgrading from reductions in misallocation.[1] We examine the role of distortions on aggregate productivity across nations using cross-country firm-level panel data and a quantitative model of misallocation featuring decisions by firms on operation (selection) and productivity-enhancing investment (technology) that impact the firm-level productivity distribution in the economy.

We construct a panel firm-level financial dataset across countries using Orbis data, collected and standardized by Bureau Van Dijk, for manufacturing firms over the period 2000-2019. Our final dataset contains 28 countries with an average of around 370 thousand firm-year observations and covers a wide range of the world income distribution. We construct a measure of firm-level total factor productivity (TFP) and a measure of firm-level distortions, a model-based measure of idiosyncratic distortions faced by the firm. Using these data we document the following facts: (1) firm-level TFP is more dispersed in less developed countries, (2) larger TFP dispersion arises mostly due to the prevalence of low productivity firms

[1]Some examples of policy reforms with effects on misallocation, technology, and selection include Pavcnik (2002) on trade reform in Chile, Bustos (2011) on technology upgrading, and Khandelwal et al. (2013) on export quotas reform in China.

in poor countries, (3) dispersion of idiosyncratic distortions is higher in less developed countries, and (4) distortions are more highly correlated with firm productivity in less developed countries. We also note that average firm size is lower in less developed countries (larger number of firms per capita) (Bento and Restuccia, 2017, 2021).

To address the quantitative role of distortions on factor misallocation and differences in the distribution of firm-level TFP across countries, we develop a model of production heterogeneity with distortions and entry and operation decisions by firms building on Hopenhayn (1992) and Restuccia and Rogerson (2008). The quantitative framework allows for productivity-enhancing investment broadly capturing costly activities that firms undertake to improve productivity or in the adoption of more advanced technologies. Production of a homogeneous good takes place in firms with access to a decreasing returns technology with labor input. Firms are subject to idiosyncratic distortions and fixed operation costs as well as a transitory productivity shock that becomes known after production decisions are made. New firms enter by paying a fixed entry cost in units of labor, after which they draw an idiosyncratic investment ability and distortion. The productivity of operating firms is determined through costly investment in which higher investment ability firms face lower investment costs and distortions affect incentives to invest. Importantly, new firms may choose not to invest or operate after drawing their idiosyncratic investment ability and distortion if their expected value is less than the fixed operating cost. This leads to selection in which less productive and more distorted firms exit the economy and are not observed in the firm distribution.

We parameterize distortions to feature a systematic component related to the elasticity of distortions to firm-level productivity, which we denote by ρ, and a random component drawn from a log normal distribution, which provide an excellent fit of measured distortions and the implied factor input allocations in the data within and across countries. We calibrate a distorted benchmark economy to micro (producer-level) and aggregate observations for France. Critical parameters are the distributions of idiosyncratic distortions and investment ability and the fixed operating costs that are jointly restricted to match moments for the French data on measured distortions, dispersion in firm-level TFP and employment, and average firm size.

While the bulk of the cross-country empirical patterns are accounted for by the model with changes in the elasticity of distortions ρ, extending the cross-country calibration to include variations in the standard deviation of the transitory component of productivity and the distortions allows the model to better fit the data in terms of the dispersion of firm-level distortions, TFP, and employment. The model is able to replicate cross-country patterns on the measured elasticity of distortions and the dispersion in the firm-level TFP, distortions, and employment. An important implication of these experiments is that the measured elasticity between firm-level TFP and distortions is biased upwards relative to the underlying population parameter and that this bias tends to be stronger in higher income countries. The bias is driven by two channels. The first, and main driver in our setting, is a selection channel in which less productive and more distorted firms exit the economy creating a mechanical positive relationship between measured TFP and distortions. Moreover, this selection channel tends to be stronger in higher income countries. The second is a noise channel in which the transitory productivity shock, which affects firm output but not inputs, creates a positive relationship between measured TFP and distortions.

We examine the quantitative effect of changes in the elasticity of distortions relative to the benchmark economy. Increasing the elasticity of distortions ρ from the calibrated value of 0.525 to 0.90, consistent with the range of values for the measured elasticity observed in the cross-country data, reduces aggregate output by 77 percent. In other words, a policy reform that reduces the elasticity of distortions from 0.90 to 0.525 as in the benchmark economy would increase aggregate output substantially by 330 percent, an increase that represents 67 percent of the aggregate labor productivity gap between Vietnam and France in our cross-country sample. We decompose the change in aggregate output in terms of the channels of resource misallocation and the change in the productivity distribution. We find that around 60 percent of the aggregate productivity loss from increasing the elasticity of distortion is due to changes in firm-level productivities, with only 40 percent resulting from static misallocation.

Allocative efficiency declines by more than static misallocation because of the change in firm-level productivities. We find that about two-thirds of the productivity loss from declining allocative efficiency is explained by a static channel in which the set of firms are held fixed and about one-third by a dynamic channel in which allocative efficiency declines due to changes in the set of operating firms and chosen technologies. These results highlight the important interaction between the firm-level productivity distribution and allocative efficiency, an underappreciated cost of misallocation in individual-country survey data. In a separate decomposition, we find that the contribution of productivity loss from the shift in the productivity distribution is roughly equally divided between selection (the change in operating producers) and technology (the investments in productivity).

Our paper closely relates to the broad literature on production heterogeneity and misallocation (Restuccia and Rogerson, 2008; Guner et al., 2008; Hsieh and Klenow, 2009) and the associated literature on producer dynamics, technology adoption, and aggregate productivity (Parente and Prescott, 1994; Bhattacharya et al., 2013; Hsieh and Klenow, 2014; Bento and Restuccia, 2017; Comin and Mestieri, 2018; Ayerst, 2022; Buera et al., 2023). We make three contributions. First, we provide a systematic assessment on the joint distribution of firm-level productivity and distortions using cross-country producer-level data. A novel finding is that the lower productivity dispersion in higher income countries is driven, in part, by a compression of the bottom-end of the productivity distribution. While there is still a large gap, low-productivity firms are much closer to high-productivity firms in high-income countries than in low-income countries. This contribution also connects us with a recent effort to exploit Orbis and other datasets to examine firm-level evidence in many countries (such as by Andrews et al., 2015; Poschke, 2018; Kalemli-Ozcan et al., 2023; Alviarez et al., 2023).

Second, we show that the cross-country facts can be reconciled by incorporating producer selection and technology investment into the standard model of misallocation. In this regard, our analysis integrates the quantitative (Restuccia and Rogerson, 2008) and empirical (Hsieh and Klenow, 2009) literatures on misallocation. The main insight is the substantial aggregate productivity effect of distortions on selection and technology, an effect missing in both of these literatures. We show that the model is able to replicate cross-country patterns both theoretically and in a calibrated quantitative model. Both selection and technology channels have been emphasized for individual countries, but the cross-country importance of these channels are typically not assessed due to lack of comparable cross-country data. We show that both the selection and technology channels are essential in reproducing the cross-country empirical patterns. Third, we derive and measure bias in estimates of the productivity elasticity of distortions, an important measure of misallocation (Restuccia and Rogerson, 2017). The bias results from selection, technology choices, and ex-post productivity shocks (mismeasurement). We find that selection is the most important source of bias in higher income countries rendering a flatter relationship between the measured elasticity and income per capita across countries. The ex-post noise bias, while quantitatively significant, represents a relatively small component of the measured elasticity, indicating that measurement error is not a major concern in the context of our cross-country data.

In a closely related paper, Fattal-Jaef (2022) uses Orbis, and other data sources, to examine the aggregate productivity costs of entry barriers using a model that also features idiosyncratic distortions, endogenous firm exit decisions, and technology investment. While both models feature selection, our focus is on the selection of (ex-ante) more productive firms into markets, in contrast to the exit of ex-post less productive firms examined by Fattal-Jaef (2022). We show that this feature of ex-ante selection is quantitatively important in accounting for the simultaneous decline in firm-level productivity dispersion and elasticity of distortions in more developed countries. The selection channel also relates us to recent papers examining the impact of misallocation on selection (Yang, 2021; Majerovitz, 2023).

The paper proceeds as follows. In the next section, we describe the data and present the main empirical observations from the cross-country data. Section 3 describes the model and characterizes the qualitative role of distortions on firm-level TFP. In Section 4, we calibrate a distorted benchmark economy to micro and aggregate data for France and quantify the effect of distortions on the distribution of firm-level TFP and other outcomes across countries. We conclude in Section 5.

2 Stylized Facts

We describe the cross-country data and provide details of constructed variables. We then present our main facts on firm-level productivity and distortions across countries.

2.1 Data

We use firm-level financial data from Orbis collected and standardized by Bureau Van Dijk as the main dataset for our analysis. Given that our goal is to characterize cross-country facts on productivity and misallocation, we focus on assembling available data for as many countries as possible in our final dataset. We restrict to countries with at least 5,000 observations after cleaning (described below). We also restrict to the period from 2000 to 2019 since earlier periods tend to have fewer observations in many countries and later periods coincide with the COVID-19 pandemic that may affect cross-firm and cross-country statistics.

Within countries, we restrict to firms in the manufacturing sector and drop firm-year observations that are missing sufficient information to construct productivity, are inactive, or are duplicate observations. We trim the remaining variables for extreme values based on output at the top and bottom 0.1% and employment greater than 100,000 workers. We drop the bottom 1% of firms based on labor share or firms where the wage bill is greater than revenue or value added. We correct employment for firm-year observations that are likely incorrectly reported by replacing employment at the top and bottom 1% of firms based on the wage bill-per-employee with the wage bill-implied employment. We also trim observations by the top and bottom 2% of the productivity and wedge (described below) distribution in each year to limit the influence of outliers. Appendix A provides a detailed description of the data and cleaning procedure.

Our final dataset contains data on 28 countries with an average of 370 thousand firm-year observations. The number of observations ranges widely across countries, with just over 5,000 observations in Mexico and Colombia and around 2 million observations in China. The dataset covers a wide range of the world income distribution with India and Vietnam among the low-income countries and France and Germany among the high-income countries.

2.2 Variable Construction

We use the data to describe the distribution of firm-level productivity and misallocation. We construct two variables that measure firm-level productivity and distortion. We refer to firm-level distortions as the firm's wedge since it is a model-based measure of the difference between the firm's realized market allocation and the hypothetical first-best allocation, in which wedges are equalized across firms. In this regard, the measure is the same as the marginal product of factor inputs in Hsieh and Klenow (2009). We derive model-based measures of productivity and wedges as:

$$\mathrm{TFP}_{i,t} = \frac{y_{i,t}}{n_{i,t}^{\gamma}}, \qquad wedge_{i,t} = \frac{y_{i,t}}{n_{i,t}}. \tag{1}$$

We construct measures of output y and employment n. We measure output as the firm operating revenue, and sales when operating revenue is unreported. We do not use value added because we find that material costs are not widely reported outside of Europe and this limits the final distribution of countries. Employment is measured as the number of employees hired by the firm. In cases where the number of employees is unavailable, we back out a measure using the wage bill of the firm and a constructed average wage rate for that firm's country sector (two-digit SIC) year.

Appendix B reports the robustness of the main results to alternative-model measures and construction of productivity and wedges in equation (1), although we note that the implied wedge in equation (1) holds in commonly-used production technologies. In particular, we show that the main cross-country observations hold if we construct total factor productivity that adjusts for capital inputs, value added as the measure of firm output, if we use a constant elasticity of substitution model as in Hsieh and Klenow (2009), or if we weight observations on the relative share of firms using national statistics data.

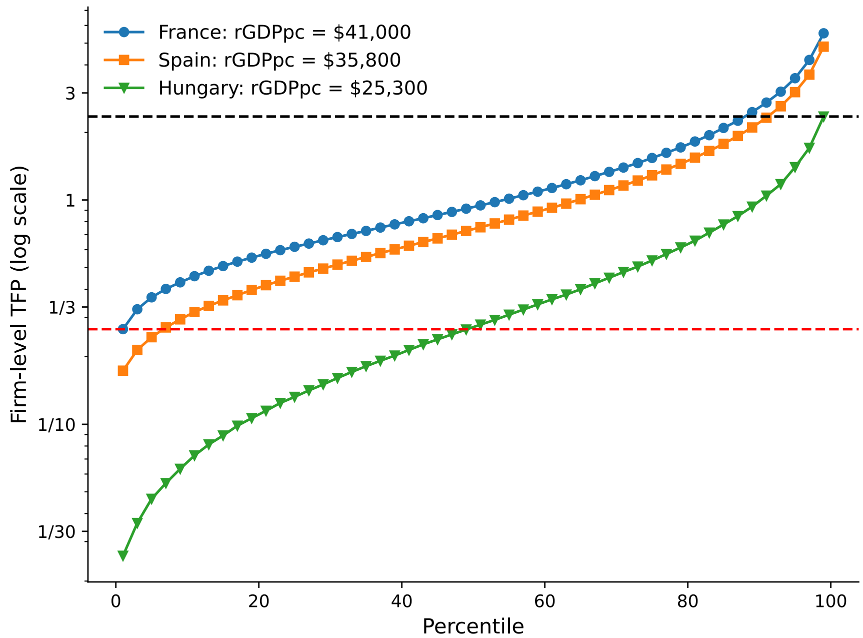
2.3 Cross-Country Productivity Distribution

We start by looking at how the firm productivity distribution varies across countries at different stages of development. For illustration, we compare the productivity distributions of three European countries in 2005 that differ in terms of the level of development. Figure 1 reports the average productivity of firms within a percentile of the productivity distribution and does not detrend productivity such that the levels of productivity are comparable across countries, where we note that all values are reported in US dollars.

Figure 1 highlights differences between the countries. First, France, a high-income country, has less dispersed productivity. Second, the productivity distributions in other countries (Spain and Hungary) appear to "fan out" from the top end of the distribution. The most productive firms in France, Spain, and even Hungary have relatively similar productivity while the productivity gap at the bottom percentile of the distribution is much larger. For instance, around 50 percent of firms in Hungary have lower TFP than the bottom one percentile firm in France, whereas slightly more than 10 percent of firms in France have higher TFP than the top percentile firm in Hungary.

Next, we use wider ranging cross-country data to draw broader conclusions on these observations. In the comparisons that follow, we demean productivity and wedges by regressing each variable on country-by-year-by-sector fixed effects. In this regard, we are comparing the

Figure 1: Productivity Distribution of Operating Firms 2005



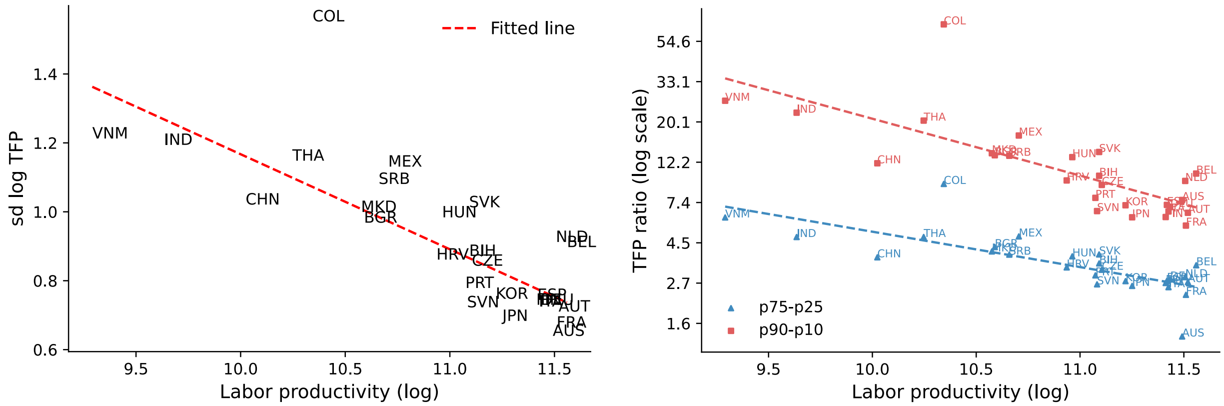
Notes: The figure reports values for 50 percentile points of the firm-level TFP distribution, from percentile one (p1) to percentile 99 (p99).

distribution of relative firm productivity excluding the level, unlike Figure 1.

Figure 2 reports the standard deviation of firm-level productivity in countries against development, measured by aggregate labor productivity in 2015 (real GDP per worker) from the Penn World Table (Feenstra et al., 2015). Data is pooled at the country level, noting that measures of productivity and wedges are demeaned in each sector-year implying that the standard deviation does not capture between-year differences. Productivity tends to be more dispersed in lower income countries. This is also confirmed by similar patterns in other measures of dispersion, such as the inter-quartile or inter-decile range, that place less weight on outliers.

Figure 3 reports the comparison of firms at different percentiles of the productivity distribution to provide information on the overall shape of the distribution across countries, and whether, in general, the productivity distribution fans out or shifts down in countries at lower levels of output per worker. We compare firms at the top of the distribution (p99) to firms at different points of the distribution (p75, p50, p10, and p1) in all countries. Noting

Figure 2: Cross-Country Productivity Dispersion



(a) Standard deviation (b) Inter-quartile, decile ranges

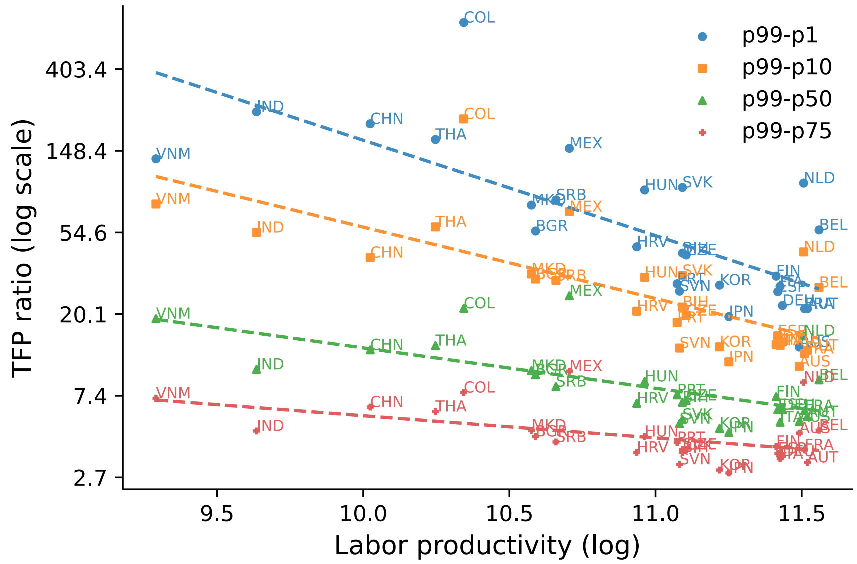
Notes: Productivity dispersion is measured by the standard deviation of log TFP across firms in each country. Each observation is the estimated value for the indicated country. Aggregate labor productivity in logs from the Penn World Table (Feenstra et al., 2015).

the log scale, a uniform shift in firm-level productivity would result in flat percentile ratios across the labor productivity distribution, while a decline in the productivity dispersion alone would result in the same slope of the four percentile ratios across the labor productivity distribution. Figure 3 confirms the pattern emphasized earlier for Spain and Hungary compared to France that the productivity distribution tends to narrow in more developed countries. This is stronger in the bottom of the distribution than in the top of the distribution. That is, the narrowing productivity distribution in more productive countries is driven more by an improvement in the relative position of less productive firms compared to firms at higher percentiles of the productivity distribution, the slope of the TFP ratio is flatter at higher points of the productivity distribution. Intuitively, this could be due to large multinational or global firms employing similar best practices occupying the top end of the distribution in most countries.

2.4 Cross-Country Wedge Distribution

We next use the data to examine how the wedge distribution varies across countries at different levels of development. We focus on two commonly reported moments: the standard

Figure 3: Comparison of Productivity Distribution



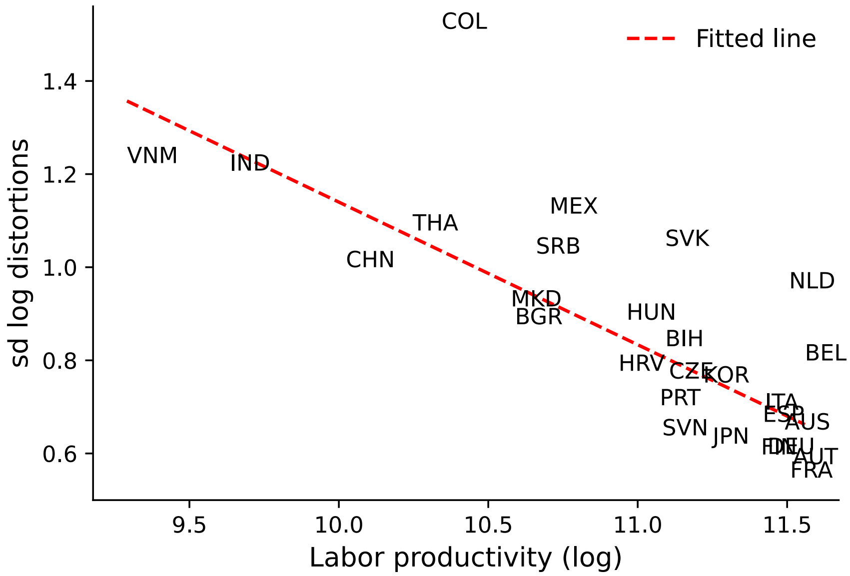
Notes: Each observation is the value for the indicated country. The dashed lines are the line of best fit. The TFP ratio in the y-axis is in log scale. Aggregate labor productivity from the Penn World Table (Feenstra et al., 2015).

deviation of wedges and the elasticity of wedges with respect to measures of productivity.

Figure 4 reports the standard deviation in the wedge across firms for all the countries in the data. Consistent with previous findings for some countries in the literature, Figure 4 shows that lower income countries tend to have more dispersed wedges.

Figure 5 reports the relationship between firm-level wedges and productivity in our cross-country data. Panel (a) reports the elasticity of wedges with respect to firm-level labor productivity. Panel (b) reports the elasticity of employment with respect to firm-level labor productivity. In both cases, lower income countries tend to be more distorted. As noted earlier, in Appendix B we show that the negative relationship between the elasticity of distortions and firm-level productivity is robust to alternative models, variable constructions, and weighting observation by firm shares. However, the specific magnitudes of these elasticities are sensitive to these choices. For example, using the Hsieh and Klenow (2009) model and parameterization, we find elasticities that are between 0.3 and 0.6, around half of the values in Figure 5. Reassuringly, for this version of the model, we find similar elasticities in

Figure 4: Cross-Country Dispersion in Distortions



Notes: Dispersion in distortions is measured by the standard deviation of log wedge across firms in each country. Each observation is the estimated value for the indicated country. Aggregate labor productivity in logs from the Penn World Table (Feenstra et al., 2015).

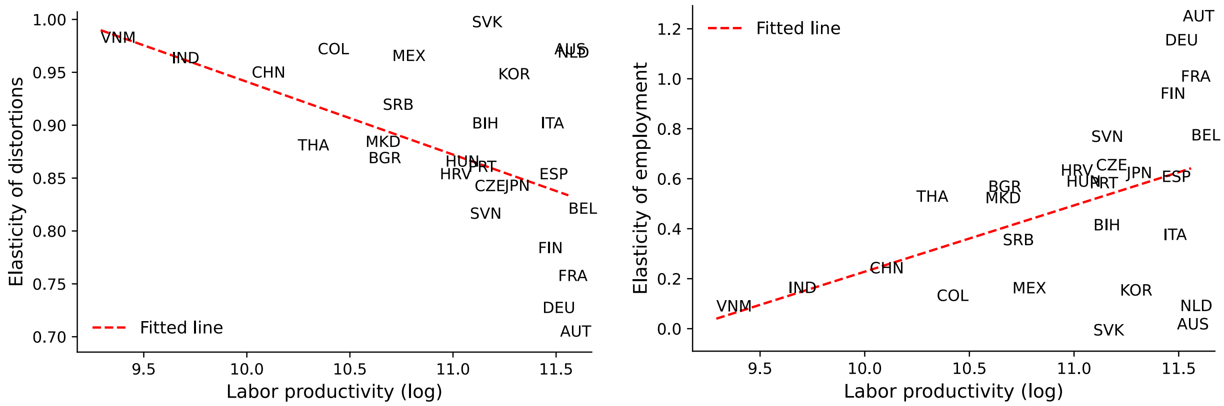
our data for China and India to those reported by Hsieh and Klenow (2009).

An important insight of our analysis is the recognition that the empirical findings on firm-level productivity and distortions may reflect empirical patterns based on firm choices on technology and in particular operation, for which only operating firms appear in the data. Analyzing these aspects of technology and selection requires more structure. In the next sections, we use our model to derive potential sources of upward bias in the measured elasticity and show that this leads to a quantitatively important gap between the measured and actual elasticity, notably in high-income countries. We also use our model to quantify the role of technology and selection in understanding differences in the firm-level productivity distribution across countries and their impact on allocative efficiency.

3 Model

We develop a model of production heterogeneity with distortions and entry and operation decisions by firms building on Hopenhayn (1992) and Restuccia and Rogerson (2008). We

Figure 5: Cross-Country Elasticity of Distortions



(a) Elasticity of distortions (b) Elasticity of employment

Notes: Elasticity of distortions measured by the slope coefficient of a regression between log wedge and log TFP in each country. Aggregate labor productivity in logs from the Penn World Table (Feenstra et al., 2015).

extend the framework to allow for productivity-enhancing investment and highlight the operation decisions of firms. We focus on a stationary competitive equilibrium of the model because our goal is to examine long-term cross-country productivity gaps.

3.1 Economic Environment

Technologies. At each date, a homogeneous good is produced by firms indexed by i. Firms have access to a decreasing-return-to-scale technology,

$$y_i = v_i {z_i}^{1-\gamma} {n_i}^{\gamma}, \quad \gamma \in (0, 1),$$

where y_i is output, n_i is the labor input, and $v_i {z_i}^{1-\gamma}$ is the firm total factor productivity. The term $z_i^{1-\gamma}$ is a permanent component of total factor productivity which is the result of a firm's investment decision while v_i is a transitory component of total factor productivity with $\mathbb{E}v_i = 1$ that is drawn each period from an iid cumulative distribution function $H(v)$ after production decisions are made (Boar et al., 2022). We note that the transitory component v_i could also capture measurement error in the data which similarly leads to a disconnect between the reported output and labor inputs.[2] Firms are subject to an operating fixed cost c_f per period in units of labor and may choose to exit, before v_i is realized, to avoid incurring this cost.

Entry and exit. In addition to the operation decision, firms exit at an exogenous rate λ every period. Entering firms incur entry cost c_e in units of labor. After paying the entry cost, firms draw a firm-specific innovation ability χ_i from an iid cumulative distribution function $G(\chi)$ and choose to invest in productivity z_i at investment cost $\psi z_i^{\phi}/\chi_i$, where $\psi > 0$ and $\phi > 1$. Firms may choose not to invest and exit the market. We denote the mass of entrants by E and the total mass of operating firms by N.

Households. There is a representative household of measure one with standard preferences on consumption $u(C) = \log(C)$. The household is endowed with one unit of productive time each period that is supplied inelastically to the market.

3.2 Market Structure

Firms face idiosyncratic distortions which we model as proportional revenue taxes τ_i as in Restuccia and Rogerson (2008). Following Bento and Restuccia (2017) and Restuccia (2019), we assume that idiosyncratic distortions feature a systematic component related with firm's productivity $z_i^{-\rho}$ and a firm-specific random component ϵ_i. Specifically, we assume that firm-level distortions $\tau_i(z_i, \epsilon_i)$ are equal to:

$$(1 - \tau(z_i, \epsilon_i)) = \left(z_i^{-\rho}\epsilon_i\right)^{1-\gamma},$$

where ρ is the elasticity of distortions with respect to the firm's permanent TFP, determining the systematic component of distortions, and ϵ_i is the random component of distortions

[2] We could also model a similar transitory component on measured labor inputs but the implications for measurement would be similar and hence for simplicity and tractability we only include the output component v_i.

drawn from an iid cumulative distribution function $F(\epsilon)$. Intuitively, ρ distorts the productivity gradient of firm size, whereas ϵ captures an effect of distortions on firm size that is independent of firm's productivity. Taxes are collected by a government that redistributes revenues as a lump-sum transfer T to households.

We model τ_i as a catch-all of the myriad of policies and institutions that affect business operation, abstracting from the specific drivers of distortions since our focus is to examine the impact of broad distortions on operation (selection) and investment (technology) decisions by firms. We emphasize, however, that the literature has identified numerous policies and institutions creating wedges in marginal products across firms in many different contexts (Hopenhayn, 2014; Restuccia and Rogerson, 2017). Prominent examples of specific policies and institutions creating systematic wedges across firms include firing taxes (Hopenhayn, 2014), financial frictions (Buera et al., 2013), size-dependent regulations (Guner et al., 2008). Similarly, numerous policy trade reforms have been shown to reduce misallocation, improve selection, and encourage technology upgrading (Pavcnik, 2002; Bustos, 2011; Khandelwal et al., 2013).

3.3 Equilibrium

We consider a stationary competitive economy in which households and firms take prices as given, prices are constant, and the distribution of resource allocations and firm types are stationary. The price of the output good is normalized to one and the price of labor is denoted by w.

Incumbent firms. An incumbent firm is characterized by productivity z and distortion τ. The firm chooses the optimal labor n to maximize expected per-period profit $\pi(z, \tau)$:

$$\begin{aligned}\pi(z_i, \tau_i) &= \max_{n \geq 0} \quad \mathbb{E}_v \left[v_i(1-\tau_i) z_i^{1-\gamma} n^{1-\gamma} - wn - c_f w \right], \\ &= \max_{n \geq 0} \quad (1-\tau_i) z_i^{1-\gamma} n^{1-\gamma} - wn - c_f w.\end{aligned}$$

In the above expression, the period transitory TFP shock v drops out of the firm problem since $\mathbb{E}v = 1$. The solution to the firm's problem implies that the labor demand and optimal output are given by

$$n(z_i, \tau_i) = (1-\tau_i)^{\frac{1}{1-\gamma}} z_i \left(\frac{\gamma}{w}\right)^{\frac{1}{1-\gamma}},$$
$$y(z_i, v_i, \tau_i) = (1-\tau_i)^{\frac{\gamma}{1-\gamma}} v_i z_i \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}}.$$

Note that given the assumed functional form for distortions, the productivity gradient of firm size is affected by ρ, with higher ρ implying a flatter size-productivity relationship.

Expected operating profits are equal to

$$\pi(z_i, \tau_i) = \Omega(1-\tau_i)^{\frac{1}{1-\gamma}} z_i - c_f w, \quad \text{where} \quad \Omega \equiv \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} (1-\gamma).$$

The expected value of a firm can be written as the expected discounted per-period profit stream. The expected value of an incumbent firm $W(z_i, \tau_i)$ is:

$$\begin{aligned} W(z_i, \tau_i) &= \max\left\{\pi(z_i, \tau_i) + (1-\lambda)\frac{W(z_i, \tau_i)}{1+r}, 0\right\}, \\ &= \max\left\{\frac{\Omega(1-\tau_i)^{\frac{1}{1-\gamma}} z_i - c_f w}{1-R}, 0\right\}, \end{aligned}$$

where $R = (1-\lambda)/(1+r)$, noting that firms with negative profit would not operate and hence have zero value. We characterize the operation decision below.

Entering firms. A firm that enters the market draws an idiosyncratic innovation ability χ_i from distribution $G(\chi)$ and the random component of the distortion ϵ_i from distribution $F(\epsilon)$. The firm then decides the level of productivity z at a cost. The firm chooses productivity to maximize the value of an incumbent firm net of productivity investment cost:

$$V(\chi_i, \epsilon_i) = \max_{z \geq 0}\left[W(z, \tau(z, \epsilon_i)) - \psi\frac{z^\phi}{\chi_i}\right],$$

where $W(z, \tau)$ is the value of an incumbent firm with productivity z and $\tau(z, \epsilon_i)$ is the distortion faced by the firm given the choice of z and the random component ϵ_i, as described above. We denote by the function $z(\chi, \epsilon)$ the optimal productivity level from this problem. Note that even though there is an optimal productivity level associated with every χ, only a fraction of firms with such χ operate in the market, a decision that depends on the random component of distortions.

Optimal productivity z for an entrant drawing (χ_i, ϵ_i) is given by:

$$z(\chi_i, \epsilon_i) = \left(\frac{(1-\rho)\tilde{\Omega}\chi_i\epsilon_i}{\psi\phi} \right)^{\frac{1}{\phi+\rho-1}}, \quad \text{where} \quad \tilde{\Omega} \equiv \frac{\Omega}{1-R}. \tag{2}$$

Note that χ and ϵ affect productivity in the same proportion and depend on the elasticity of distortions ρ.

Using this optimal productivity and substituting for the value of an incumbent firm, the value of an entrant firm drawing (χ_i, ϵ_i) is given by:

$$\begin{aligned} V(\chi_i, \epsilon_i) &= \max \left\{ \tilde{\Omega} z(\chi_i, \epsilon_i)^{1-\rho}\epsilon_i - \psi \frac{z(\chi_i, \epsilon_i)^{\phi}}{\chi_i} - \frac{c_f w}{1-R}, 0 \right\}, \\ &= \max \left\{ \Gamma(w, \rho)\chi_i^{\frac{1-\rho}{\phi+\rho-1}} \epsilon_i^{\frac{\phi}{\phi+\rho-1}} - \frac{c_f w}{1-R}, 0 \right\}, \end{aligned}$$

where

$$\Gamma(w, \rho) \equiv \frac{\phi+\rho-1}{\phi} \tilde{\Omega} \left(\frac{(1-\rho)\tilde{\Omega}}{\psi\phi} \right)^{\frac{1-\rho}{\phi+\rho-1}}.$$

As firms only operate when their value is non-negative, the decision to operate for a firm drawing (χ_i, ϵ_i) can be characterized as:

$$o(\chi_i, \epsilon_i) = \begin{cases} 1 & \text{if} \quad \Gamma(w, \rho)\chi_i^{\frac{1-\rho}{\phi+\rho-1}} \epsilon_i^{\frac{\phi}{\phi+\rho-1}} \geq \frac{c_f w}{1-R}, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

At the beginning of each period, the entry value V_e is given by,

$$V_e = \mathbb{E}_{\chi,\epsilon} V(\chi, \epsilon) - c_e w \leq 0.$$

The entry condition requires that potential entrants enter to the point where further entry is no longer valuable, and is equal to zero in an equilibrium with positive entry.

Firm distribution. The firm distribution is straightforward to characterize since we abstract from firm dynamics other than entry, exit, and the random productivity shock v. In particular, the firm distribution over productivity levels can be determined from the distribution of firms over innovation ability χ and the random distortion ϵ. We denote by $\mu(\chi, \epsilon)$ the mass of producers over firm types. The law of motion for $\mu(\chi, \epsilon)$ is given by:

$$\mu'(\chi, \epsilon) = (1 - \lambda)\mu(\chi, \epsilon) + Eo(\chi, \epsilon)dF(\epsilon)dG(\chi),$$

which implies that in a stationary equilibrium where the distribution of firms is constant. The stationary distribution is given by:

$$\mu(\chi, \epsilon) = \frac{E}{\lambda} o(\chi, \epsilon)dF(\epsilon)dG(\chi). \tag{4}$$

The mass (number) of firms in a stationary equilibrium is

$$N = \int_\chi \int_\epsilon d\mu(\chi, \epsilon) = \frac{E}{\lambda} \int_\chi \int_\epsilon o(\chi, \epsilon)dF(\epsilon)dG(\chi). \tag{5}$$

Definition of equilibrium. A stationary competitive equilibrium comprises a wage w; decision functions for firms: labor demand $n(z, \tau)$, profits $\pi(z, \tau)$, value of incumbent firm $W(z, \tau)$, productivity $z(\chi, \epsilon)$, operating decision $o(\chi, \epsilon)$, net value of firm $V(\chi, \epsilon)$, value of entry V_e, a distribution of firms $\mu(\chi, \epsilon)$, mass of firms N and entrants E; lump-sum transfer T; and allocation C for households such that:

(i) Given w and T, the allocation C solves the household's problem.

(ii) Given w, decision function $n(z,\tau)$ solves the incumbent's firm problem, determining per-period profits $\pi(z,\tau)$ and the value of incumbent firms $W(z,\tau)$.

(iii) Given w, entrants choose productivity $z(\chi,\epsilon)$ and operating decision $o(\chi,\epsilon)$ to maximize the net value of the firm $V(\chi,\epsilon)$.

(iv) Zero profit entry condition $V_e = 0$.

(v) Invariant distribution of firms μ given by equation (4), which implies the mass of firms is constant and given by equation (5).

(vi) The government's budget is balanced:

$$0 = T + \int_\chi \int_\epsilon \tau(\chi,\epsilon)(1-\tau(\chi,\epsilon))^{\frac{\gamma}{1-\gamma}} z(\chi,\epsilon)\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} \mu(d\chi,d\epsilon).$$

(vii) The goods and labor markets clear:

$$\int_\chi \int_\epsilon z(\chi,\epsilon)^{1-\gamma} n(\chi,\epsilon)^\gamma \mu(d\chi,d\epsilon) = C + E\int_\chi \int_\epsilon \psi \frac{z(\chi,\epsilon)^\phi}{\chi} o(\chi,\epsilon) dG(\chi) dF(\epsilon),$$

and

$$1 = \int_\chi \int_\epsilon n(\chi,\epsilon) o(\chi,\epsilon)\mu(d\chi,d\epsilon) + Ec_e + Nc_f.$$

Equilibrium solution. The stationary competitive equilibrium is straightforward to compute. Given a wage rate w, all firm decision functions can be solved and since V_e is a strictly decreasing function of w, the zero profit entry condition solves for w (see Proposition 1). The labor market clearing condition solves for the mass of entry E which in turn determines all other variables such as the invariant distribution and number of firms.

Proposition 1. *The equilibrium wage rate is determined by the zero profit entry condition:*

$$\int_{\chi}\int_{\epsilon}\max\left\{\Gamma(w,\rho)\chi_i^{\frac{1-\rho}{\phi+\rho-1}}\epsilon_i^{\frac{\phi}{\phi+\rho-1}}-\frac{c_f w}{1-R},0\right\}G(d\chi)F(d\epsilon)-c_e w=0. \tag{6}$$

The equilibrium wage rate w is decreasing in the elasticity of distortions ρ.

The left-hand side (LHS) of equation (6) represents the expected value of potential entrants which must be zero in an equilibrium with positive entry as in our framework. Since $\Gamma(w,\rho)$ is decreasing in w and ρ, the LHS is a decreasing function of w and ρ. Given a w, when ρ increases, the LHS decreases. As a result, when ρ increases, w has to decrease for the zero profit entry condition to hold.

3.4 Model Implications

In the empirical section, we emphasized the distributional properties of firm-level productivity and wedges across countries. We now discuss how the model relates with these observations. The measured firm-level productivity and wedge of firm i in the model are given by:

$$\text{TFP}_i=\frac{y_i}{n_i^{\gamma}}=z(\chi_i,\epsilon_i)^{1-\gamma}v_i, \tag{7}$$

$$wedge_i=\frac{y_i}{n_i}=\left(\frac{w}{\gamma}\right)\frac{v_i}{1-\tau_i}. \tag{8}$$

The above expressions show the relationship between measured productivity and wedges. The ex-post productivity v_i implies a mechanical relationship between measured productivity and wedges that we show below impacts the relationship between the model and data. Additionally, technology choice z implies that firm-level measures of TFP may capture misallocation of talent in which distortions impact the relationship between firm ability and measured productivity.

Technology and selection both have important implications for the measurement of firm productivities and wedges. On technology, measured productivity in equation (7) varies endogenously with firm technology choices z. Firm technology choice from equation (2) can be rewritten as:

$$\log\left(z(\chi_i, \epsilon_i)\right) = \frac{1}{\phi+\rho-1}\log\left(\frac{(1-\rho)\tilde{\Omega}}{w\psi\phi}\right) + \frac{1}{\phi+\rho-1}\left[\log(\chi_i)+\log(\epsilon_i)\right]. \tag{9}$$

We are interested in characterizing how technology choice z, which is a function of innovation ability χ, is affected by distortions. First, as noted earlier, the random component of distortions ϵ affects technology choice in the same proportion as χ. Hence, this factor alone generates some dispersion in z for a given χ. Second, technology choice is affected by the systematic component of distortions in two ways. The first term in equation (9) represents a constant in the relationship between technology choice z and χ that captures the general equilibrium impact of distortions and productivity on average technology investment. The second term represents the gradient of χ differences on technology choice. A higher elasticity of distortions ρ, lowers the χ-gradient of technology choice, reducing differences in technology choice across firms. Hence, on this factor, the model would imply lower dispersion of firm-level TFP in higher ρ economies, in contrast to our empirical fact across countries in development.

On selection, the cutoff condition for the operation decision of firms in equation (3) can be written as:

$$\frac{1-\rho}{\phi+\rho-1}\log(\chi) + \frac{\phi}{\phi+\rho-1}\log(\epsilon) \geq \log\left[\frac{c_f w}{(1+R)\Gamma(w,\rho)}\right]. \tag{10}$$

The left-hand side of this equation is decreasing in ρ, whereas the effect on the right-hand side (RHS) is ambiguous. Hence, the effect on selection is ambiguous, depending on the quantitative impacts of ρ and w. It follows from the above expression that a higher RHS (more selection) implies that, for a given χ, operating firms have a higher random component of distortions ϵ (equivalently, a lower τ). In turn, this implies that higher selection is associated with more negative measured covariance between χ and ϵ, which we denote by $cov(\chi, \epsilon|o)$, as well as lower variances of χ and ϵ conditional on operating, denoted as $\sigma^2_{\chi|o}$ and $\sigma^2_{\epsilon|o}$. Propositions 2 and 3 highlight the impact of these factors on productivity dispersion and the productivity elasticity of distortions in economies that vary in ρ.

Proposition 2. *Dispersion in productivity across firms is given by:*

$$\begin{aligned} var(TFP) &= (1-\gamma)^2\sigma^2_{z|o} + \sigma^2_v, \\ &= \left(\frac{1-\gamma}{\phi+\rho-1}\right)^2 (\sigma^2_{\chi|o} + \sigma^2_{\epsilon|o} + 2\ cov(\chi, \epsilon|o)) + \sigma^2_v. \end{aligned}$$

Productivity dispersion is decreasing in the elasticity of distortions ρ and in the extent of selection.

Proposition 2 shows that the measured variance of TFP across firms is decreasing in the elasticity of distortions ρ, other things equal. In the absence of selection (i.e., when $cov(\chi, \epsilon|o) = 0$, $\sigma^2_{\chi|o} = \sigma^2_\chi$, and $\sigma^2_{\epsilon|o} = \sigma^2_\epsilon$), more distorted economies have lower productivity dispersion, in contrast with the data. Our empirical facts point to stronger selection in higher income countries, offsetting the impact of technology choice on the variance of measured productivity.

Proposition 3. *The elasticity of measured wedges with respect to productivity is given by:*

$$elas(TFP, wedge) = \frac{\rho(1-\gamma)^2\sigma^2_{z|o} + \sigma^2_v - (1-\gamma)^2 cov(z, \epsilon|o)}{(1-\gamma)^2\sigma^2_{z|o} + \sigma^2_v}. \tag{11}$$

Proposition 3 shows three biases in the estimated elasticity of measured productivity and wedges. First, the ex-post random component of productivity v creates a mechanical relationship between the measured wedge and productivity that increases the elasticity. Since firms cannot adjust inputs in response to this shock, the shock has the same impact on both the measured wedge and productivity creating a positive bias. Second, selection decreases the covariance of firm-level productivity and wedges creating a positive bias in the estimated elasticity. Intuitively, this is because unproductive (low χ), high distortion (high τ) firms select out of the economy, making it more likely that observed relatively unproductive firms have relatively low distortions (low τ), biasing upwards the measured elasticity. Third, firms choose technology z based on their draw of random distortions ϵ creating a mechanical negative relationship between measured wedges and productivity, biasing downwards the estimated elasticity. For intuition, in an economy with $\chi = 1$ for all firms and elasticity of distortions $\rho = 0$, higher ϵ firms (lower τ) would choose more productive technologies z (positive $cov(z, \epsilon|o)$) leading to a downward bias on the measured elasticity of distortions (see also, Ayerst, 2022).

It follows that the measured elasticity of productivity and wedges accurately reflect the underlying elasticity of distortions ρ only when these three biases are zero, such that firms have full ex-ante information ($\sigma_v^2 = 0$), there is no selection ($cov(\chi, \epsilon|o) = 0$), and technology is exogenous. The actual magnitude of these biases and their net impact across countries is a quantitative question that we examine in detail in the next section.

4 Quantitative Analysis

We proceed in three steps. First, we calibrate a distorted benchmark economy to micro and aggregate data for France. Second, we show that quantitatively plausible differences in distortions can explain cross-country differences in the data moments documented in Section 2. Even after accounting for measurement bias, the TFP elasticity of distortions accounts for the bulk of cross-country differences. Third, we decompose the productivity losses from varying distortions across economies into its components of static misallocation, selection, and technology.

4.1 Calibration

We calibrate a distorted benchmark economy to micro and aggregate data for France. We parameterize the distributions of $\log \chi$, $\log v$, and $\log \epsilon$ to be normal with normalized means and standard deviations σ_χ, σ_v, and σ_ϵ, respectively. There are 11 parameters to calibrate in the model: the decreasing returns to scale γ, the exogenous firm exit rate λ, the real interest rate r, the dispersion in innovation ability σ_χ, the dispersion transitory ex-post productivity shock σ_v, the level and curvature parameters of innovation cost function ϕ and ψ, the fixed costs of entry c_e and operation c_f, the productivity elasticity of distortions ρ, and the dispersion of the random wedge component σ_ϵ.

A set of 6 parameters are either normalized or assigned values from outside evidence. We set the decreasing returns to scale to $\gamma = 0.8$ as is commonly used in the misallocation literature (Guner et al., 2008; Restuccia and Rogerson, 2008), the exit rate to $\lambda = 0.10$ (Davis et al., 1998), the real interest rate to $r = 0.04$, the curvature of investment cost function to $\phi = 2$ (Acemoglu et al., 2018). We normalize the productivity investment cost $\psi = 1$ and the cost of entry $c_e = 1$.

The remaining five parameters ρ, σ_ϵ, σ_χ, σ_v, and c_f are jointly calibrated to match the following moments from the French firm-level data: (1) the distortion-productivity elasticity, (2) the standard deviation of log distortions, (3) the standard deviation of log employment, (4) the standard deviation of log TFP, and (5) average firm size.

Table 1: Calibration of Distorted Benchmark Economy

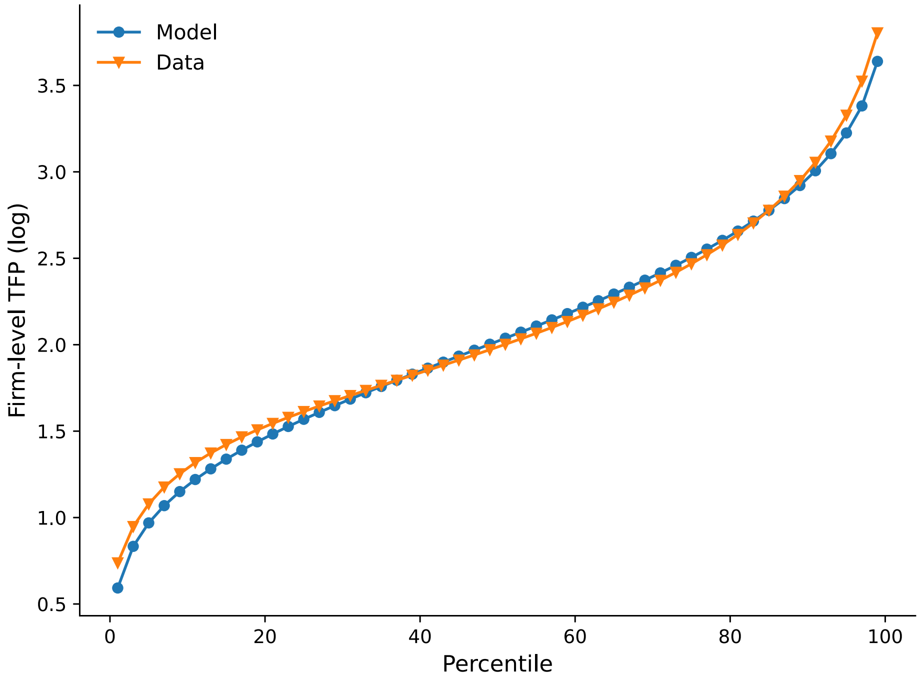
Parameter	Value	Targeted moments	Model	Data
ρ	0.525	Measured elasticity of distortions	0.75	0.75
σ_ϵ	1.4	sd log distortions	0.55	0.55
σ_χ	11.0	sd log employment	1.31	1.31
σ_v	0.2	sd log TFP	0.68	0.66
c_f	0.14	Average firm size	14.7	14.9

Table 1 reports the calibrated parameter values and the model and data moments. We note that the calibrated parameter values for distortions ($\rho = 0.525$ and $\sigma_\epsilon = 1.4$) imply values of distortion moments that are consistent with estimates found in other studies (Hsieh and Klenow, 2009). We also note that the calibrated parameter $\rho = 0.525$ implies substantial bias in the measured elasticity of distortions of 0.75. As discussed in the model, the upward bias in the elasticity is due to selection of operating firms and the mechanical correlation created by the ex-post random component of productivity v. We later show that the selection channel accounts for the majority of the bias in the benchmark economy.

Figure 6 reports the implied percentile distribution of firm-level TFP in the model compared to the French data. Despite the calibration assuming a log normal distribution of innovation abilities and only targeting the standard deviation of log TFP, the resulting distribution of firm-level TFP matches closely to that of the French data. The calibrated benchmark economy features strong selection of firms in operation, such that many potential firms do not operate. For instance, the percentile 1 operating firm by TFP would be the percentile 83 firm if all firms were to operate.

We also highlight how the calibration moments provide identification for the model parameters. Table 2 reports the resulting changes in the model implied moments from a 10 percent increase in each calibrated parameter, highlighting the extent of interconnectedness between the model moments and parameters. The measured elasticity of distortions is most sensitive to the model parameter ρ, but also depends on the other parameters due to direct or indirect impacts of these parameters on the bias associated with the magnitude of this measured moment. The standard deviation of distortions mostly reflects the model distortions themselves through ρ and σ_ϵ, but is also impacted by ex-post productivity dispersion σ_v because this is measured as part of the wedge, and by the other parameters through their impact on productivity dispersion and selection. The dispersion parameters σ_v and σ_χ and the fixed cost c_f all directly impact either the dispersion in potential productivity or firm productivity types that select into operating, which impacts the dispersion in firm productivity and employment as well as the average firm size. Similarly, the distortion parameters ρ and σ_ϵ directly impact the dispersion of firm productivity and employment and the mass

Figure 6: Firm-level TFP Distribution in Benchmark Economy and France



Notes: Data refers to the distribution of firm-level TFP in France from Orbis, whereas Model refers to the calibrated distribution of productivity in the distorted benchmark economy. For ease of illustration, the figure only plots 50 percentile points of the distributions, from percentile one (p1) to percentile 99 (p99). Means of the distributions are normalized to be equal.

of firms through firm technology and operating choices.

4.2 Cross-Country Experiments

We consider the model's fit and ability of calibrated distortions to explain the cross-country moments documented in Section 2. To parameterize the set of cross-country economies, we recalibrate the model to match moments for Vietnam, the country with the lowest aggregate labor productivity in our data. Similar to France, we find that the model is able to replicate the data moments for Vietnam and that the resulting firm-level TFP distribution matches the empirical distribution in Vietnam quite well (further details are provided in Appendix D.1). We also find that calibrated values of the dispersion of innovation abilities σ_χ and the fixed operating costs c_f are relatively similar to those in the benchmark economy. Since our goal is to assess cross-country differences arising from distortions, we hold σ_χ and c_f fixed

Table 2: Effects of 10% Changes in Calibrated Parameter Values

	ρ	σ_ϵ	σ_v	σ_χ	c_f
Measured elasticity of distortions	4.8	1.8	0.6	-0.9	0.3
sd log distortions	9.2	4.3	1.4	2.2	-0.3
sd log TFP	6.0	2.4	0.9	3.2	-0.6
sd log employment	-6.0	1.3	0.0	3.7	-0.9
Average firm size	-26.2	9.0	-0.9	17.7	5.3

Notes: The values indicate the percent changes in the moment when the indicated parameter is increased by 10 percent relative to the benchmark value and all other parameters are fixed at benchmark values.

in the cross-country experiments.

We report model values for economies with parameters $(\rho, \sigma_\epsilon.\sigma_v)$ on a gradient of intermediate values between the France and Vietnam calibrations, as reported in Table 3. We also divide the experiments into cross-country economies that correspond to the benchmark economy where one of the three sets of parameters are adjusted: (1) the elasticity of distortions $(\Delta\rho)$; (2) the elasticity of distortions and the standard deviation of distortions $(\Delta\rho, \sigma_\epsilon)$; and (3) the elasticity of distortions, the standard deviation of distortions, and the dispersion of the ex-post productivity component $(\Delta\rho, \sigma_\epsilon, \sigma_v)$. The aim of these experiments is to provide a measure of the relative importance of distortions and mismeasurement in accounting for cross-country differences.

Table 3: Values of ρ, σ_ϵ, and σ_v for Cross-Country Experiments

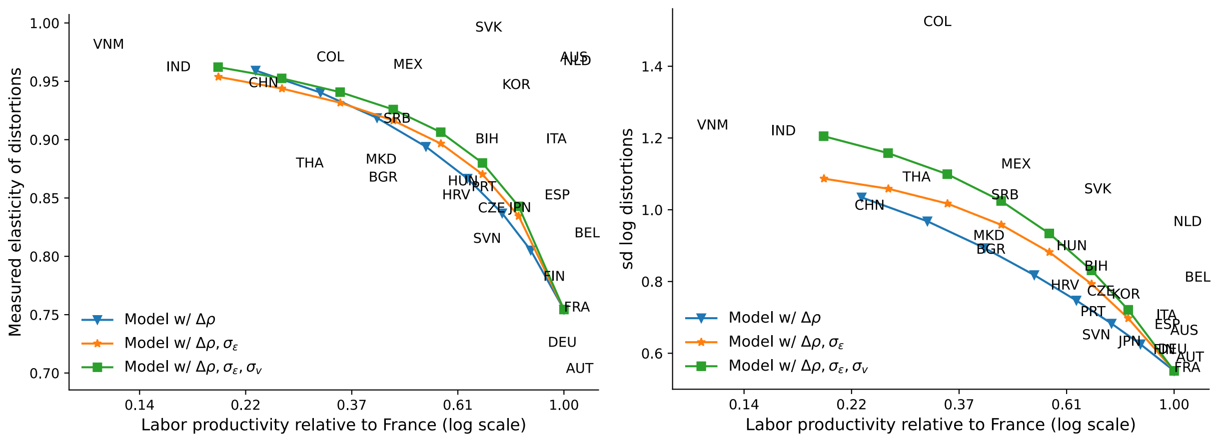
ρ	0.525	0.60	0.65	0.70	0.75	0.80	0.85	0.90
σ_ϵ	1.40	1.84	2.13	2.43	2.72	3.01	3.31	3.60
σ_v	0.20	0.27	0.32	0.37	0.42	0.47	0.51	0.56

Model fit. We compare the cross-country results for moments of interest against the cross-country data from Section 2. For each moment, we report the implications against aggregate labor productivity in the data and the model.[3] In the model, aggregate labor productivity is

[3]We avoid using cross-country measures of aggregate TFP because of the important measurement issues associated with this statistic in the data, especially related to measures of capital, but we emphasize that

simply aggregate output since aggregate labor is constant across economies. We also note that we report model moments against the implied aggregate labor productivity. An implication is that aggregate labor productivity in the model calibrated to Vietnam is higher than the data for Vietnam. The model under-accounts for the France-Vietnam labor productivity gap, which was not targeted in the calibration.

Figure 7: Elasticity and Standard Deviation of Distortions



(a) Elasticity of distortions (b) Standard deviation of log distortions

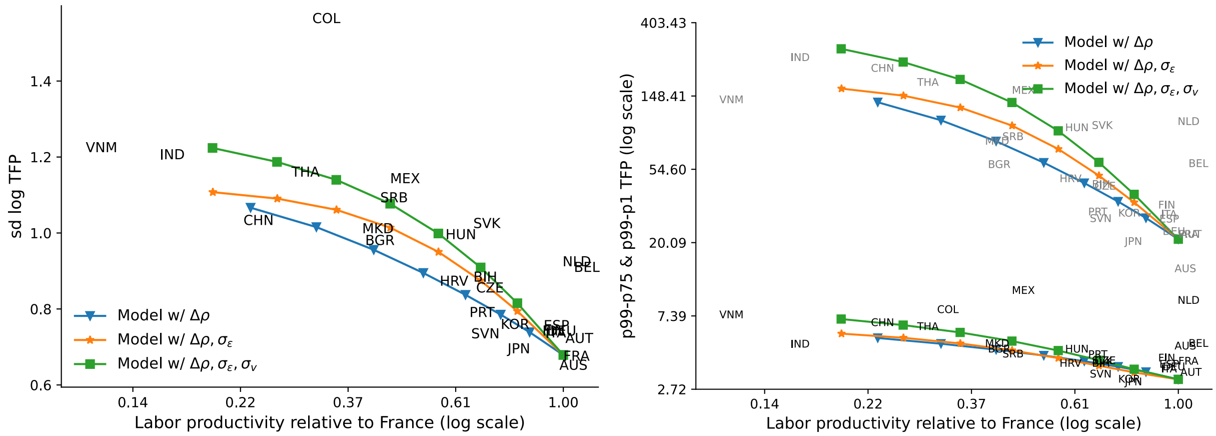
Notes: The triangle-blue line represents the model implied moment with only variation in ρ. The star-orange and square-green lines report the same moments but for economics that also differ on σ_ϵ, and on σ_ϵ and σ_v, respectively, according to the values reported in Table 3.

Figure 7 documents the relationship between the measured elasticity and standard deviation of distortions and aggregate labor productivity across countries. Three important patterns arise. First, the three models fit the cross-country data relatively well. Second, the bulk of the empirical relationship between distortions and aggregate labor productivity is generated by differences in the model elasticity of distortions ρ across economies, even after accounting for potential biases in the measured elasticity. Third, differences in aggregate labor productivity in the model represent more than 67% of the variation in the cross-country data. For instance, in the data Vietnam is about 11% of the aggregate labor productivity in France, whereas in the model the most distorted economy features aggregate labor productivity that is 23% of the benchmark economy when only ρ varies, and 20% when σ_v and

our main conclusions would hold.

σ_ϵ also vary. This implies that aggregate output differences in the model represent between 67% ($\log(0.23)/\log(0.11)$) and 73% ($\log(0.20)/\log(0.11)$) of the variation in the cross-country data.

Figure 8: Dispersion Measures of Firm-Level TFP



(a) Standard deviation of log TFP

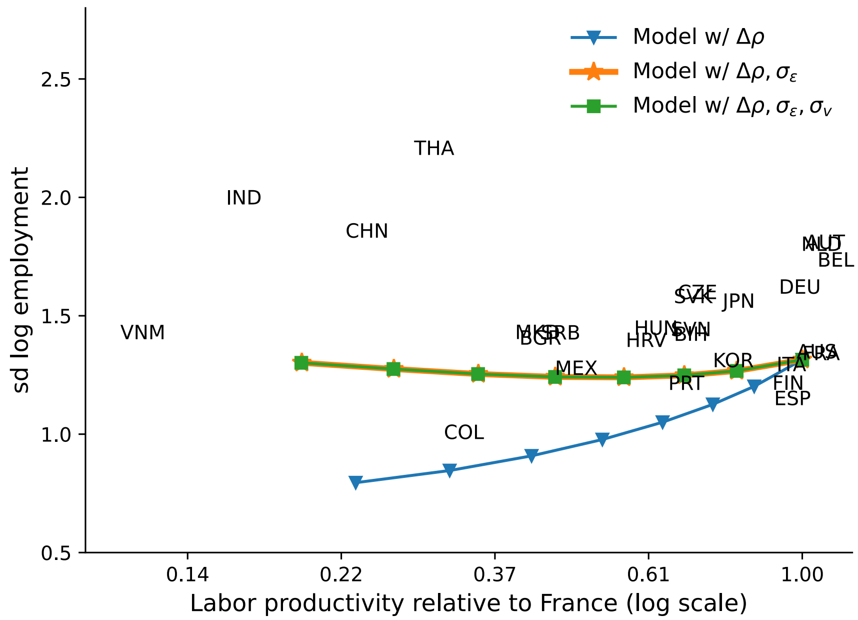
(b) p99-p75 & p99-p1 of log TFP

Notes: The blue-triangular, orange-starred, and green-squared lines correspond to experiments: (1) varying ρ, (2) varying ρ and σ_ϵ, and (3) varying ρ, σ_ϵ, and σ_v, respectively, according to values reported in Table 3. Panel (b) reports the ratio p99/p75 and p99/p1 against aggregate labor productivity.

Figure 8 reports the changes in the firm-level TFP distribution across countries as reported by both the standard deviation of log TFP and the gaps between the p99 and p75 and p99 and p1 firms. We exclude other gaps reported in Figure 3 for clarity. As with the distortions, all three models fit the data relatively well given that, at most, three parameters are being adjusted and the model elasticity of distortions accounts for the bulk of the cross-country differences. While the model elasticity ρ explains the bulk of cross-country differences, the inclusion of the standard deviations of distortions σ_ϵ and ex-post productivity shock σ_v become more important in accounting for the observed dispersion in the standard deviation of firm-level TFP in middle and low labor productivity countries. We also note that the p99 and p75 and p99 and p1 gaps are not targeted in any part of the calibration and so the model's quantitative fit acts a goodness-of-fit check on the theory and calibration.

Figure 9 reports the dispersion in firm-level employment, measured by the standard deviation of log employment across countries. The data does not indicate a systematic relationship

Figure 9: Employment Dispersion across Firms

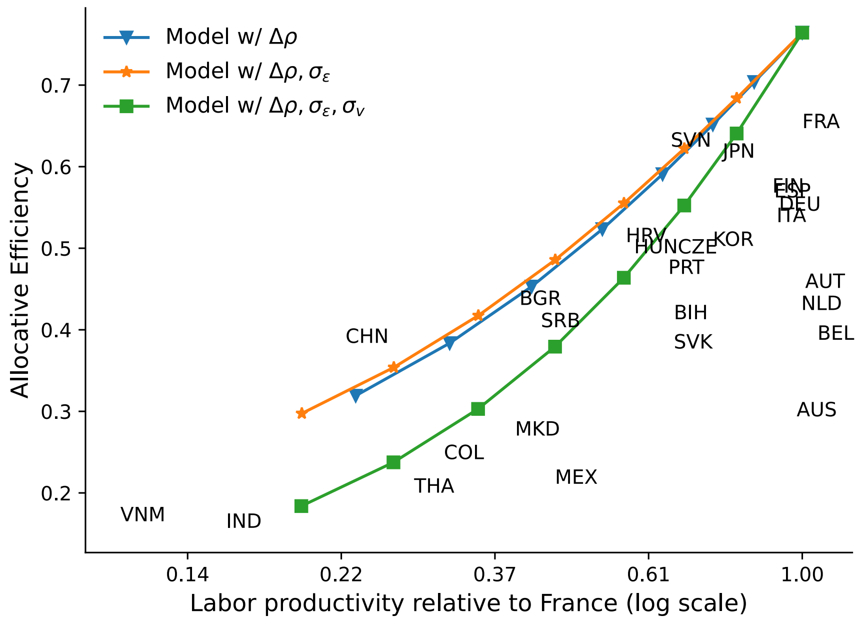


Notes: Employment dispersion is measured by the standard deviation of log employment across firms. The blue-triangular, orange-starred, and green-squared lines correspond to experiments: (1) varying ρ, (2) varying ρ and σ_ϵ, and (3) varying ρ, σ_ϵ, and σ_v, respectively, according to values reported in Table 4. Experiments (2) and (3) are perfectly aligned because v does not affect employment decisions.

between the dispersion in firm size and aggregate productivity across countries. Variation in the elasticity of distortions ρ alone results in lower dispersion in firm size in higher ρ economies since higher values of ρ directly compress the employment distribution. Allowing for variation in σ_ϵ realigns the model with the data. This suggests that the dispersion in firm size is driven more by productivity dispersion in low elasticity ρ economies, where employment is closer to efficient levels, and more by the dispersion of distortions in less developed countries. In Appendix D, we show that a model with selection or technology alone could not replicate the relatively flat profile of cross-country employment dispersion. In this regard, the dispersion in employment moment also serves as a check on the need to account for both technology choice and selection.

Figure 10 reports allocative efficiency in the model experiments and the data. Allocative efficiency is the ratio of aggregate output in an economy to the aggregate efficient output that could be achieved if labor were efficiently allocated, for the given set of operating

Figure 10: Allocative Efficiency



Notes: The triangle-blue line represent the allocative efficiency in the model with only variation in ρ. The star-orange and square-green lines report the same but for economics that also differ on σ_ϵ, and σ_ϵ and σ_v, respectively, according to the values reported in Table 4.

firms and technologies in each economy. As a result, allocative efficiency is a measure of misallocation that combines the impact of factor misallocation with potential differences in the productivity distribution across economies.

While our calibration does not target allocative efficiency, the model implies reasonable levels and ranges of allocative efficiency across countries, an important additional check on the distributional implications of distortions on firm-level TFP. The model implies an allocative efficiency of 0.76 in the benchmark economy, close to to 0.65 allocative efficiency in the data for France, whereas the model with variation in ρ, σ_ϵ and σ_v implies allocative efficiency of 0.19, close to the 0.18 allocative efficiency in Vietnam. More generally, the model fits well with the measured allocative efficiency across countries in the data, albeit higher in some cases, which is to be expected given the stylized nature of the model and assumed distributions.

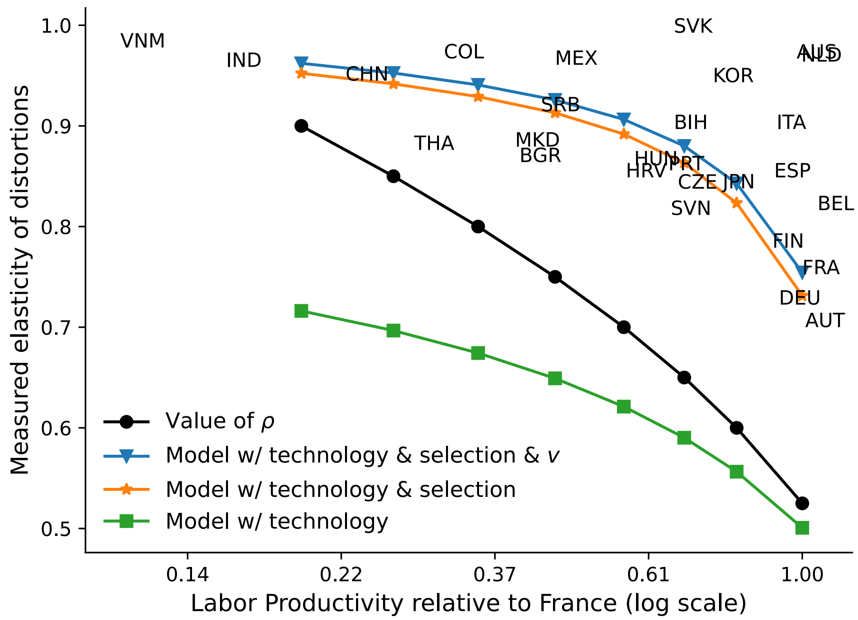
The model also generates similar gaps in allocative efficiency across countries. For example, the gap in allocative efficiency between France and India is around 50 percentage points in the data, whereas the model with only variation in ρ generates a gap of around 44 percentage points between the benchmark economy and the $\rho = 0.90$ economy. We also note that unlike the quantitative effect on aggregate output, changes in allocative efficiency as a measure of the cost of misallocation are more susceptible to mismeasurement such as potential cross-country variation in σ_v.

Estimation bias. We close the cross-country discussion by examining the estimated biases for the measured elasticity of distortions, as described in Proposition 3. Given the importance of the elasticity of distortions in the literature (e.g., Adamopoulos and Restuccia, 2014; Bento and Restuccia, 2017; Fattal-Jaef, 2022; Ayerst, 2022; Ayerst et al., 2023), we view an important contribution of our work to provide a systematic assessment on the size of this bias in existing empirical studies.

Our model implies three sources of bias in the estimated elasticity of measured productivity and distortions: the ex-post random component of productivity v, selection, and endogeneity due to the choice of technology. Importantly, given that the ex-post productivity source captures gaps between firm-level output and input decisions, it would include potential measurement error. Figure 11 decomposes the bias between the measured elasticity of distortions calculated in the data and the estimated model parameter ρ. Comparing the measured elasticity of distortions (Model w/ technology & selection & v) in the full model and the parameter values of ρ indicates substantial bias in the measured elasticity. The estimated bias ranges from just over 5% in the Vietnam calibration ($= 0.96/0.90 - 1$) to around 50% in the France calibration ($= 0.75/0.525 - 1$).

In addition to the full model, we consider two alternative models to decompose the sources of estimation bias: (1) the full model shutting down ex-post productivity shocks v (Model w/ technology & selection), and (2) the full model shutting down both ex-post productivity shocks v and the selection channel by allowing all firms to operate (Model w/ technology). Shutting down the ex-post productivity shocks v reduces the estimation bias. The magni-

Figure 11: Estimation Bias in Measured Elasticity of Distortions



Notes: The circle-black line represents the values of ρ across experiments and what would be measured if there was no bias. The triangle-blue line represent the measured elasticity of distortions in the model with variation in ρ, σ_ϵ and σ_v, respectively, according to the values reported in Table 3. The star-orange report the measured elasticity of distortions assuming no productivity shock v across experiments. The square-green lines report the same assuming not productivity shock v and no selection of operating firms across experiments.

tudes suggest a limited upward bias in estimating the distortion elasticity due to ex-post productivity shocks, including measurement error. Additionally, the bias resulting from the ex-post productivity shock v uniformly affects different economies in the experiment.

Shutting down both ex-post productivity shocks v and the selection channel results in a smaller measured elasticity than the value of ρ, resulting in a downward bias. This is consistent with the theoretical prediction discussed in Proposition 3 that endogeneity in technology choice creates a downward bias in the estimate of the elasticity of distortions (see also, Ayerst, 2022). The experiment suggests that the main driver of the upward bias in our context is the selection channel of operating firms. Stronger selection of operations in less distorted economies also creates more upward bias in estimating the elasticity of distortions.

4.3 Decomposing Productivity Losses

The previous experiments highlight that the model can account for key cross-country moments, that there are substantial cross-country losses stemming from distortions, and that the calibrated elasticity of distortions ρ accounts for the bulk of cross-country differences, even after correcting for several sources of measurement bias. We now use the benchmark economy, calibrated to France, to provide insights into the sources of productivity losses.

We focus on increasing the elasticity ρ from the benchmark value of 0.525 to 0.65, 0.80 and 0.90, consistent with the implied cross-country range of measured elasticities. We also report an economy with $\rho = 0$ for reference, although we note that this economy is not an undistorted economy since the random component of distortions σ_ϵ is fixed at its calibrated value in the benchmark economy in all the experiments.

Table 4 reports the results. The top row documents aggregate output in each economy relative to the benchmark $\rho = 0.525$ economy. Recall that because total labor is constant across economies, the change in aggregate output represents a change in labor productivity. Increasing ρ from 0.525 to 0.90 reduces aggregate output substantially by 77 percent, from a normalized value of 1.00 in the benchmark economy to 0.23 in the $\rho = 0.90$ economy. Put differently, if the $\rho = 0.90$ economy were to implement policy reforms to reduce ρ to 0.525 as in the benchmark economy, aggregate output would increase in this economy by 4.3-fold (an increase of 330 percent).

We are interested in decomposing the large change in aggregate output into its sources and to relate with existing approaches in the literature. We start with the most narrow form of misallocation, which we denote by static misallocation in the second row of Table 4, as the effect of increased distortions in the benchmark economy. That is, the effect on aggregate output of higher ρ among the same set of producers as in the benchmark economy. This type of misallocation is the focus in Restuccia and Rogerson (2008) and a large quantitative literature. Static misallocation generates a reduction in aggregate output of 45 percent. Hence, static misallocation accounts for about 41 percent ($= \log(0.55)/\log(0.23)$) of the total

Table 4: Experiments with Alternative ρ Values

	Value of ρ				
	0.00	0.525	0.65	0.80	0.90
Aggregate output	1.49	1.00	0.75	0.41	0.23
A. Static versus dynamic misallocation					
Static misallocation	1.09	1.00	0.88	0.69	0.55
Contribution (%)	*22*	–	*44*	*42*	*41*
Dynamic misallocation					
Firm-level productivity	1.34	1.00	0.88	0.70	0.56
Contribution (%)	*73*	–	*44*	*40*	*40*
Firm productivity with distortions	1.02	1.00	0.97	0.86	0.77
Contribution (%)	*5*	–	*12*	*18*	*19*
Allocative efficiency	1.11	1.00	0.85	0.59	0.42
Contribution (%)	*27*	–	*56*	*60*	*60*
B. Technology versus selection					
Technical efficiency	2.38	1.00	0.76	0.52	0.38
Technology	1.38	1.00	0.88	0.72	0.58
Contribution (%)	*37*	–	*46*	*52*	*58*
Selection	1.72	1.00	0.86	0.73	0.68
Contribution (%)	*63*	-	*54*	*48*	*42*

Notes: Static misallocation is aggregate output with distortions when operating firms i and technologies z_i are held fixed at the benchmark economy. Firm-level productivity is aggregate output in the efficient allocation in each economy. Allocative efficiency is aggregate output relative to efficient aggregate output. Technical efficiency is the average of firm-level productivity in the efficient allocation. Technology is technical efficiency keeping the operation decisions of firms constant at the benchmark economy, whereas Selection is calculated as a residual from technical efficiency and technology.

loss in aggregate output in the $\rho = 0.90$ economy. The remaining 59 percent results from changes in firm-level productivities. The direct effect of changes in firm-level productivity associated with increased distortions can be measured as the change in aggregate efficient output since in the efficient allocation this measure only depends on firm-level productivities in each economy. Relative to the benchmark economy, the change in the distribution of firm-level productivity in the third row in Table 4 reduces aggregate output by 44 percent, accounting for 40 percent (log 0.56/log 0.23) of the loss in aggregate output in the $\rho = 0.90$

economy. The remaining 19 percent is an interaction of changes in firm-level productivities with increased distortions (fourth row in Table 4), noting that this form of misallocation would not arise without changes in the productivity distribution. Intuitively, distortions that tend to compress employment among producers such as increased ρ have larger effects on aggregate output when the set of low productivity producers is larger.

We also report allocative efficiency in Table 4. Recall that allocative efficiency is measured in each economy as aggregate output divided by aggregate output under the efficient allocation among the given set of operating firms and technologies in each economy. The inverse of allocative efficiency is the efficiency gain and is the measure of misallocation emphasized in Hsieh and Klenow (2009) using plant-level data for China, India, and the United States. Note that allocative efficiency, which can be calculated with data without the need of much structure, differs from static misallocation precisely when the set of producers and technologies differ across economies, an analysis that requires more structure. Consistent with our finding of substantial changes in the distribution of firm-level productivity, allocative efficiency drops much more than static misallocation, from 0.76 in the benchmark economy to 0.32 in the $\rho = 0.90$ economy, representing about 60 percent ($= \log(0.42)/\log(0.23)$) of the loss in aggregate output. This finding helps rationalize why quantitative analyses of misallocation when producer productivity is constant tend to find much smaller effects than empirical analyses of allocative efficiency (Restuccia and Rogerson, 2017).

Our framework can be used to decompose allocative efficiency into a static misallocation channel that measures allocative efficiency for a fixed set of producers and a dynamic misallocation channel that measures the changes in allocative efficiency due to changes in the set and technologies of producers. Since static misallocation accounts for about 40 percent of the loss in aggregate output and allocative efficiency for 60 percent, we conclude that this static component is two-thirds of the loss attributed to allocative efficiency, with the remaining one-third due to the change in the productivity distribution. In this regard, we note that allocative efficiency as a measure of the cost of misallocation only captures a fraction of the total effect of distortions on aggregate output since this measure only captures the indirect effect of changes in the productivity distribution.

Taken together, changes in the firm-level TFP distribution account for 60 percent of the overall decline in aggregate output (direct and indirect) and 40 percent arising from increased static misallocation. This implies that the dynamic channels of selection and technology investment represent one and a half times the productivity loss from static misallocation alone, consistent with other evidence on the dynamic impacts of misallocation (e.g., Ayerst, 2022; Ayerst et al., 2023).

It is of interest to further decompose the contribution to the change in technical efficiency (the shift in the productivity distribution) arising from the selection channel (operation decision of firms) versus the technology channel (productivity-enhancing investment decision of firms). To do so, we calculate technical efficiency in each economy assuming that the set of operating firms is the same as in the benchmark economy. That is, we calculate technical efficiency using the operation decision function $o(\chi, \epsilon)$ of the benchmark economy to control for selection differences. We report this counterfactual in Panel B of Table 4. We find that the effect on technical efficiency is roughly equally shared between selection and technology channels. However, note that for smaller values of ρ the role of technology differences is smaller and hence the contribution of selection is larger.

To summarize, plausible variations in the elasticity of distortions ρ from the benchmark economy generate implications on misallocation and firm-level TFP distributions consistent with the cross-country data, including a variation in aggregate output that is 1.5-fold larger than that from static misalloction alone.

5 Conclusions

We examine the disparity in aggregate productivity across nations using cross-country firm-level panel data and a quantitative model of misallocation featuring decisions by firms on operation (selection) and productivity-enhancing investment (technology). Empirically, we find that less developed countries feature higher distortions and larger dispersion in firm-level productivity, mostly resulting from the prevalence of unproductive firms compared to developed countries. Quantitatively, we find that the aggregate productivity cost of misallocation extends beyond static misallocation. Measured distortions in the form of higher productivity elasticity of distortions generate large aggregate output losses, 60 percent of which are accounted for by changes in the productivity distribution. About one-third of the change in allocative efficiency is attributed to the change in misallocation due to changes in the productivity distribution and the remaining two-thirds is static misallocation.

Our quantitative analysis provides a connection of policies and institutions known to generate static misallocation to account for much lower levels of allocative efficiency and large aggregate output losses in more-distorted less-developed countries. Our analysis provides a parsimonious modeling of changes in firm-level productivity. Further work is needed to investigate the specific channels that may be important in accounting for productivity differences such as differential management practices, lags in technology diffusion, barriers to foreign multinationals, as well as the specific drivers of selection into market operation. Similarly, more work is needed to identify the specific policies that are relevant in accounting for the distortion patterns in less developed countries, an analysis that may require more specific country contexts. Of particular interest would be macroeconomic studies that connect specific policy or institutional reforms with empirical effects on misallocation, selection, and technology.

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Online Appendix

A Data Details

We describe the details of the construction for the final dataset.

A.1 Variables, Sample Selection, and Data Cleaning

Our final dataset is constructed over the period 2000 to 2019 and observations are at the firm-year (i, t) level. In most countries, the number of observations increases substantially around 2000 and starts to decline in more recent periods. We additionally drop 2020 and later periods to avoid including the COVID-19 pandemic. We use two-digit SIC codes (denoted by s) as our definition for the firm's sector, which we base on the firm's primary sector of operations. Kalemli-Ozcan et al. (2023) discuss the advantages and disadvantages of the Orbis dataset and provide a comparison of aggregate outcomes with national statistics.

Variables. Our baseline model requires us to construct measures of firm-level output $y_{i,t}$ and labor $n_{i,t}$. We measure output as the firms reported operating revenues, or sales when operating revenues is unavailable. We use this measure instead value added since material costs are not systematically available in most non-European countries in our dataset. We measure employment by the reported count of employees at the firm. We also use capital $k_{i,t}$ to construct a measure of firm-level TFP in a robustness analysis. We measure capital as the total book value of firm fixed assets. For some robustness checks, we also construct measures of firm-level value added, as an alternative measure of output, in which we subtract material costs from operating revenue (or sales) data.

For firm-year observations that are missing employment data, we construct employment using the wage bill. Firm we construct the average wage bill of firms within a sector year as $\bar{w}_{s,t} = \sum_{i \in \mathcal{I}_{s,t}} W_{i,t}/n_{i,t}$ for firms that report both the wage bill $W_{i,t}$ and employment, where

$\mathcal{I}_{s,t}$ be the set of firms in sector s and year t. We use $\bar{w}_{i,t}$ to construct firm employment as the reported wage bill divided by the constructed wage rate, i.e., $\hat{n}_{i,t} = W_{i,t}/\bar{w}_{i,t}$. For firm-year observations with both wage bill and employee data, we replace the top and bottom 1% of employee count by the wage bill-implied employment to reduce the likelihood that outliers are driven by misreporting.

Dropped observations. We drop observations based on the following criteria:

- **Missing data.** We drop firm-year observations without sufficient data to construct our baseline measures of productivity and wedges that require information on sales and employment. We also consider alternative models that require capital and material costs as well as data on the previous and next period variables.

- **Inactive firms.** We exclude firms that are listed with unknown status, in bankruptcy, dissolved, in liquidation, or inactive in the current period and each following period.

- **Sectors.** We focus on the manufacturing sector and only include firms with four-digit NACE code between 1000 and 3300. We also exclude firms identified as a non-corporate identity (e.g., bank).

- **Data trimming.** We trim the top and bottom 0.1% of firm observations within a year based on output, firms with more than 100,000 employees, and firms that have a reported labor share (labor cost divided by output) of more than one or in the bottom 1% of the sample. We trim the top and bottom 2% based on the measure of productivity and wedges after removing year and sector differences, such that the trimming does not target specific sectors or years. In our productivity measures that include capital, we also drop observations where the capital-output and capital-labor ratios are in the top or bottom 1% of the sample.

Multiple observations. Many firms report multiple filings within a year for various reasons. We remove multiple observations based on:

- **Consolidated financial records**. Firms may report financial records for either unconsolidated, consolidated, or both. In the case where both are reported, we default to using the unconsolidated records.
- **Filing type**. Firms may report financial records as "annual reports" or "local registry filings". In the case where both are reported, we use the annual reports.
- **Other duplicates**. Other instances of firms reporting multiple filings are relatively rare and for the most part represent duplicated data. For these duplicates, we choose between the maximum and minimum observed values based on which values minimize the absolute error with output and employment in the previous period.

Time and sector trends. We regress each nominal variable on year-by-sector fixed effects and report summary statistics for the residualized variable. That is, for variable $\tilde{X}_{f,t}$ we estimate $\log \tilde{X}_{f,t} = \Gamma_{s,t} + \log X_{f,t}$ and then construct the detrended variable as $\log X_{i,t} = \log \tilde{X}_{i,t} - \Gamma_{s,t}$.

A.2 Firm Weights

An issue with the Orbis data is that it tends to over-sample large firms and under-sample small firms in some countries. Kalemli-Ozcan et al. (2023) report that European countries tend to reflect the size distribution of firms reported in Eurostat (based on national statistics). However, less is known about the coverage outside of European countries. We construct firm weights using national statistics to allow us to re-weight the data to match the true distribution. We show that our results are robust to this re-weighting in Appendix B.

We denote our final firm weights as $\omega_{n,t}$ and denote $h^{D}_{[\underline{n},\overline{n}],c,t}$ as the share of firms with between $\underline{n}$ and $\overline{n}$ employees from dataset D. For example, the share of firms in France in 2013 with between 10 and 19 employees in the Orbis dataset is denoted by $h^{Orbis}_{[10,19],FR,2013}$. We construct the final firm weights as

$$\omega_{[\underline{n},\overline{n}],c,t} = \frac{h^{D}_{[\underline{n},\overline{n}],c,t}}{h^{Orbis}_{[\underline{n},\overline{n}],c,t}}.$$

We discuss the construction of $h^{D}_{[\underline{n},\overline{n}],c,t}$ using nationally representative data below.

Eurostat data. We use the distribution of firms by employment size as reported by Eurostat in the business demography (BD) and structural business statistics (SBS) datasets to construct observation weights. The BD dataset reports more granular data for smaller business sizes and separates non-employer businesses. The BD dataset reports firms in employment bins $\{0, 1-4, 5-9, 10+\}$. The SBS dataset is more granular at higher employment levels but lumps non-employers into the smallest size bin. The SBS dataset reports firms in employment bins $\{0-9, 10-19, 20-49, 50-249, 250+\}$.

We exclude non-employer businesses from the final dataset. We construct the final Eurostat bins as:

$$\begin{aligned}
h^{ES}_{[1,4],c,t} &= \frac{h^{SBS}_{[0,9],c,t} - h^{BD}_{0,c,t}}{1 - h^{BD}_{0,c,t}} \times \frac{h^{BD}_{[1,4],c,t}}{h^{BD}_{[1,4],c,t} + h^{BD}_{[5,9],c,t}}, \\
h^{ES}_{[5,9],c,t} &= \frac{h^{SBS}_{[0,9],c,t} - h^{BD}_{0,c,t}}{1 - h^{BD}_{0,c,t}} \times \frac{h^{BD}_{[5,9],c,t}}{h^{BD}_{[1,4],c,t} + h^{BD}_{[5,9],c,t}}, \\
h^{ES}_{[10,19],c,t} &= \frac{h^{SBS}_{[10,19],c,t}}{1 - h^{BD}_{0,c,t}}, \\
h^{ES}_{[20,49],c,t} &= \frac{h^{SBS}_{[20,49],c,t}}{1 - h^{BD}_{0,c,t}}, \\
h^{ES}_{[50,249],c,t} &= \frac{h^{SBS}_{[20,49],c,t}}{1 - h^{BD}_{0,c,t}}, \\
h^{ES}_{[250,\infty],c,t} &= \frac{h^{SBS}_{[250,\infty)}}{1 - h^{BD}_{0,c,t}}.
\end{aligned}$$

We extend the firm shares $h^{ES}_{[\underline{n},\overline{n}],c,t}$ to earlier and late periods by assuming that firm shares are the same as in the closest period. For example, if the earliest period with sufficient data for Austria is 2005 then we assume the weights $h^{ES}_{[\underline{n},\overline{n}],AT,2005}$ also apply to the period 2000 to 2004. Firm shares tend to be relatively stable over time and this allows us to maximize the usable data. We also interpolate data missing in intermediate periods as a linear combination of the two surrounding periods.

OECD data. The OECD database reports the firm size distribution divided into either three or five size bins. The three size bin categories are $\{1-19, 20-249, 250+\}$ and the five size bin categories are $\{1-9, 10-19, 20-49, 50-249, 250+\}$. We follow same procedure as with the Eurostat data to fill in missing data.

We also use the OECD data to construct weights for countries without alternative sources. For these countries, we first construct the expected share of firms in the $20-249$ size bin by regressing $h_{[20,249],c,t} = \alpha \ln GDP/Capita_{c,t} + F_t + \epsilon_{c,t}$, where F_t is a year fixed effect. The coefficient α captures the relationship between a countries output per worker and the size distribution, where $\alpha > 0$ ($\alpha < 0$) implies that wealthier countries have more (fewer) firms in this size bin.

Individual country data. We supplement the above information with data on Vietnam, Mexico, and Korea. The Vietnam data, from the Vietnamese Statistical Yearbook, groups firms into the size categories $\{1-4, 5-9, 10-49, 50-199, 200-299, 300+\}$ and is available from 2004 to 2019. The Mexico data groups firms into the size categories $\{1-10, 11-50, 51-250, 250+\}$ and is available every five years between 2004 to 2019. The Korea data groups firms into the the size categories $\{1-4, 5-9, 10-49, 50-99, 100-199, 200-299, 300+\}$ and is available from 2011 to 2019.

A.3 Overview of Final Dataset

Table A.1 reports the number of observations in the final dataset along with the source of the firm distribution used to construct firm weights. We drop countries without at least 5,000 observations after the previously described cleaning and trimming is done in order to reduce sample size issues. That said, countries have a wide range of observations in the final dataset from just over 5,000 in Colombia, Montenegro, and Mexico to almost 2 million observations in China.

Table A.1: Final Dataset

Country	Observations	Firm Distribution (Source)
Austria (AUT)	27,657	Eurostat
Australia (AUS)	12,567	OECD
Bosnia and Herzegovina (BIH)	46,971	Eurostat
Belgium (BEL)	113,690	Eurostat
Bulgaria (BGR)	156,530	Eurostat
China (CHN)	1,987,483	Estimate
Colombia (COL)	6,169	Estimate
Czech Republic (CZE)	168,581	Eurostat
Germany (DEU)	195,840	Eurostat
Spain (ESP)	1,262,738	Eurostat
Finland (FIN)	143,123	Eurostat
France (FRA)	1,046,480	Eurostat
Croatia (HRV)	127,810	Eurostat
Hungary (HUN)	309,065	Eurostat
India (IND)	138,793	Estimate
Italy (ITL)	1,675,006	Eurostat
Japan (JPN)	537,463	OECD
Korea (KOR)	1,195,930	Statistics Korea
Mexico (MEX)	6,834	Mexico Economic Census
Montenegro (MNE)	6,527	Estimate
North Macedonia (MKD)	34,438	Eurostat
Netherlands (NLD)	15,987	Eurostat
Portugal (PRT)	414,366	Eurostat
Serbia (SRB)	171,442	OECD
Slovenia (SVN)	90,740	Eurostat
Slovakia (SVK)	11,956	Eurostat
Thailand (THA)	11,956	Estimate
Vietnam (VNM)	128,837	Vietnam Statistical Yearbook

B Empirical Analysis

We examine the robustness of the main empirical results to alternative constructions of productivity and wedges. In addition, for each version of the model, we compare outcomes with labor input (tfp^{lo}) and a Cobb-Douglas aggregate of capital and labor (tfp^{cd}).

- **Value added**: The baseline results use gross output to construct statistics since this improves the representation across countries. In the alternative model, we construct output as sales $s_{i,t}$ subtract material costs $m_{i,t}$. The measures of productivity are then:

$$tfp^{lo}_{i,t} = \frac{s_{i,t} - m_{i,t}}{n^{\gamma}_{i,t}}, \qquad tfp^{cd}_{i,t} = \frac{s_{i,t} - m_{i,t}}{(k^{\alpha}_{i,t} n^{1-\alpha}_{i,t})^{\gamma}}.$$

- **Constant elasticity of substitution**: Hsieh and Klenow (2009) construct a model in which firms have constant returns to scale and face constant elasticity of substitution against products produced by other firms. In this version of the model, productivity can be constructed as

$$tfp^{lo}_{i,t} = \frac{(p_{i,t} y_{i,t})^{\frac{\sigma}{\sigma-1}}}{n_{i,t}}, \qquad tfp^{cd}_{i,t} = \frac{(p_{i,t} y_{i,t})^{\frac{\sigma}{\sigma-1}}}{k^{\alpha}_{i,t} n^{1-\alpha}_{i,t}}.$$

- **Population weighting**: The final version of the model that we report is identical to the baseline model but we weight the results by the population weights constructed in Appendix A.

Wedges are the same in each version of the model since wedges do not rely on the structure of the production function. We construct wedges based on the Cobb-Douglas inputs, the labor input, and the capital input. In the case where the distortion is on firm revenues, as opposed to factor inputs, these three wedges are theoretically equivalent. This is not necessarily the case when distortions tend to impact one factor more than the other, such as, if credit constraints limit capital inputs more than employment.

$$wedge^{y}_{i,t} = \frac{y_{i,t}}{k^{\alpha}_{i,t}\ell^{1-\alpha}_{i,t}}, \qquad wedge^{\ell}_{i,t} = \frac{y_{i,t}}{\ell_{i,t}}, \qquad wedge^{k}_{i,t} = \frac{y_{i,t}}{k_{i,t}}.$$

where in the value added approach the numerator is sales minus material costs.

B.1 Firm Productivity Distribution

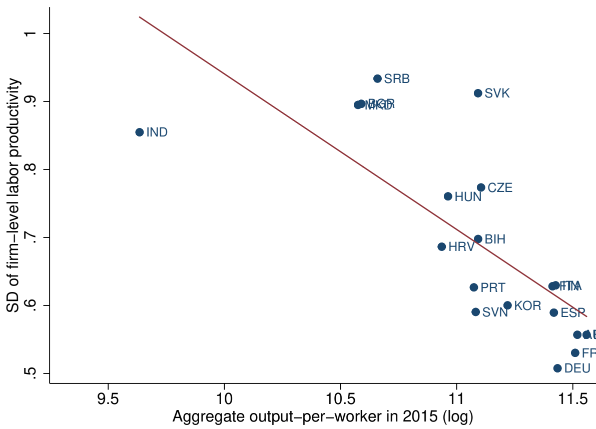
Figure B.1 reports the standard deviation of firm productivity across the six different models described above. We find a similar decreasing pattern across all six models as in the main text. In the value added measures there are fewer lower income countries included in the sample because material costs are not reported in these countries. In general, the Cobb-Douglas production function tends to reduce some of the variation in productivity as it controls for cross-firm differences in capital intensity. The CES assumption on the production and demand functions implies higher productivity dispersion. While Hsieh and Klenow (2009) show that this model is isomorphic to our baseline model, the two models make different assumptions on the mapping of data to productivity, potentially explaining the differences. We find that weighting observations increases the measured dispersion of productivity, which could reflect larger dispersion in productivity of small, under-sampled firms.

Figure B.2 reports the p99 to p75 ratio and p99 to p1 ratio for each of the six models. As in the baseline results, we find that in all cases there is more of a fanning out of the top end of the productivity distribution at lower labor productivity countries. Similar to the previous set of figures, the magnitude of the results depends on the model used, with the CES model having a notably much larger gap in the percentile ratios.

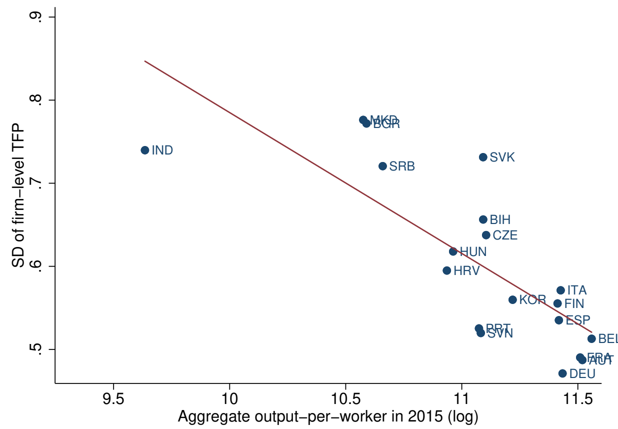
B.2 Elasticity of Wedges

Figure B.3 reports the elasticity of distortions with respect to the constructed measure of firm-level productivity in each of the six models. As with the baseline results, we find that

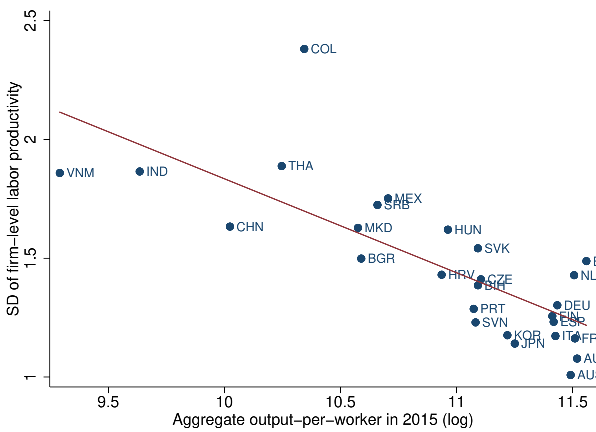
Figure B.1: Standard Deviation of Firm Productivity



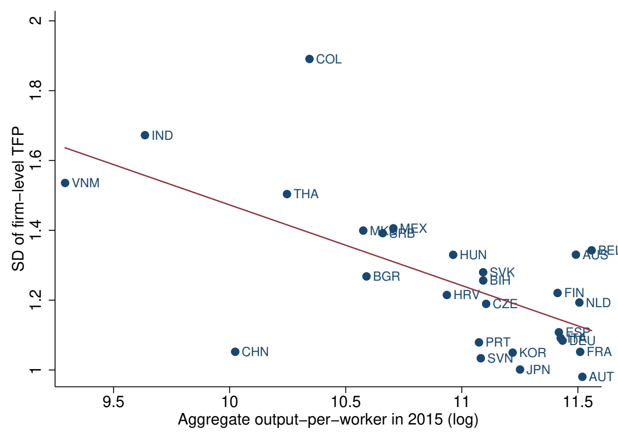
(a) Value added labor only tfp^{lo}



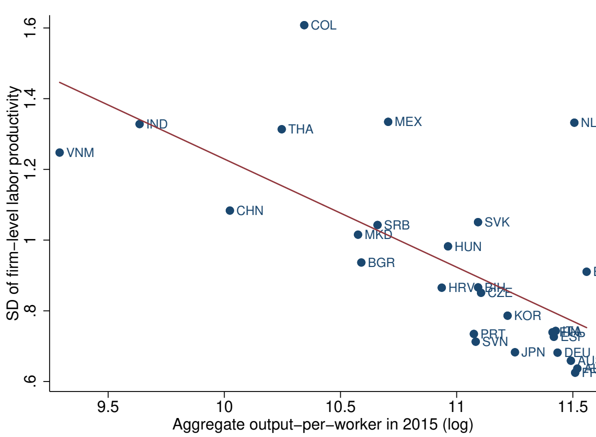
(b) Value added Cobb-Douglas tfp^{cd}



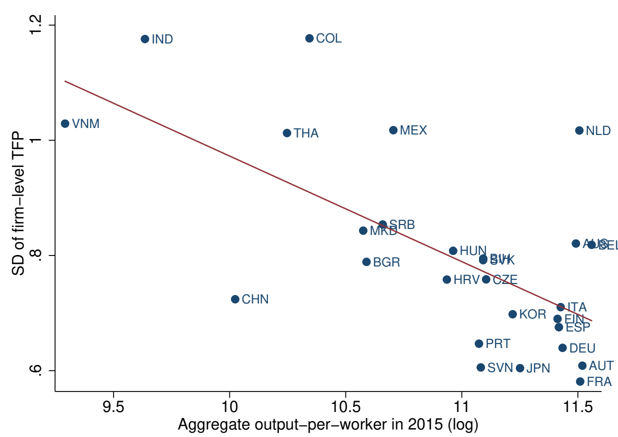
(c) CES labor only tfp^{lo}



(d) CES Cobb-Douglas tfp^{cd}

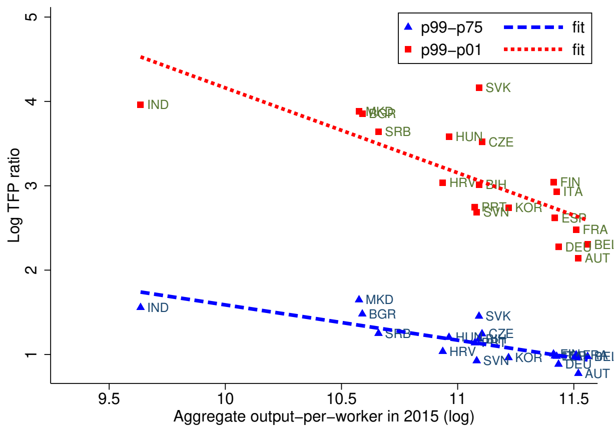


(e) Weighted labor only tfp^{lo}

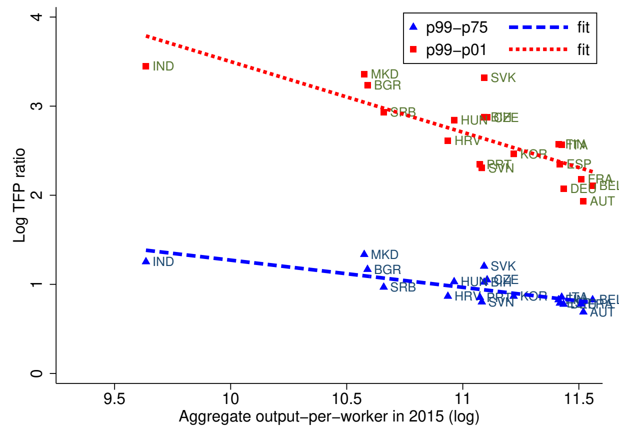


(f) Weighted Cobb-Douglas tfp^{cd}

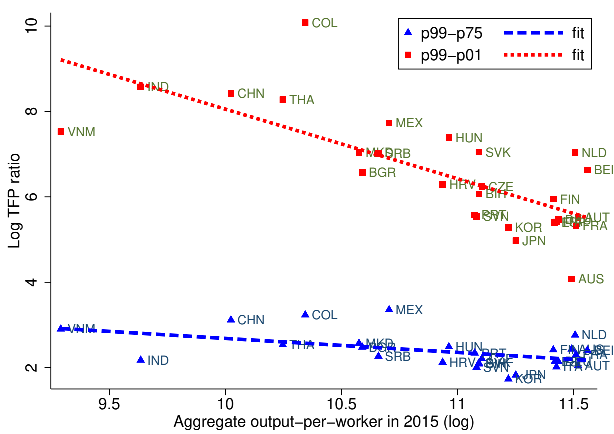
Figure B.2: Firm Productivity Distribution



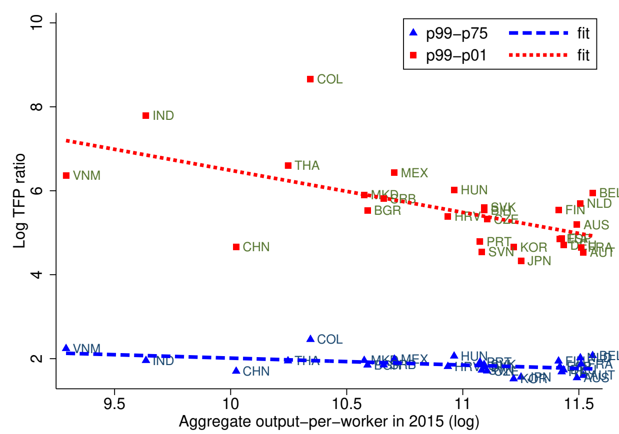
(a) Value added labor only tfp^{lo}



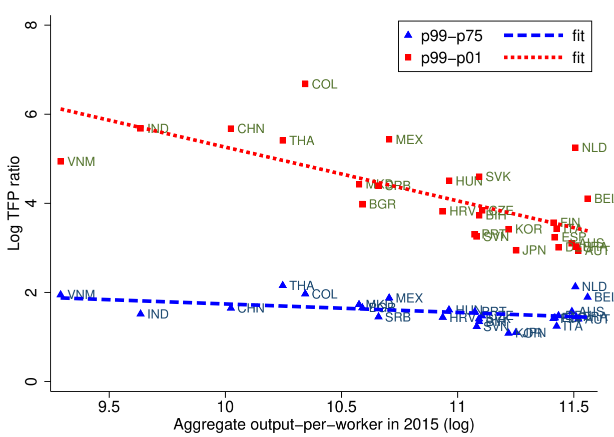
(b) Value added Cobb-Douglas tfp^{cd}



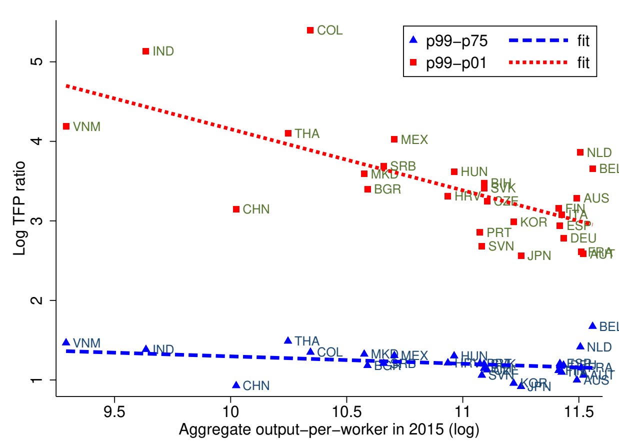
(c) CES labor only tfp^{lo}



(d) CES Cobb-Douglas tfp^{cd}



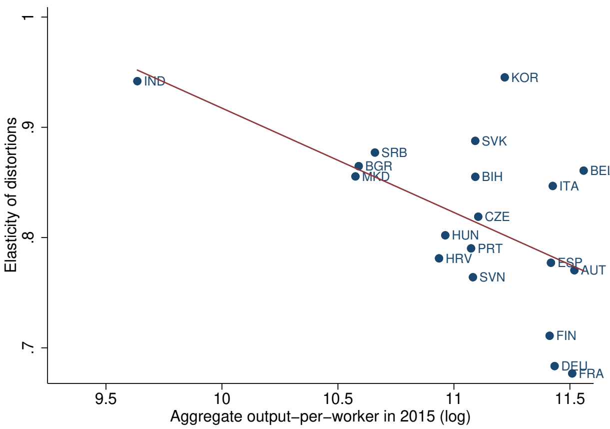
(e) Weighted labor only tfp^{lo}



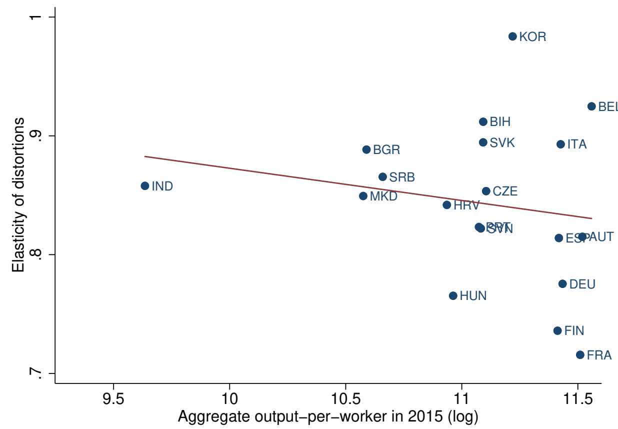
(f) Weighted Cobb-Douglas tfp^{cd}

the higher income countries tend to have lower measured elasticity of distortions. We find that this moment is sensitive to the choice of model with values ranging between 0.3 and 0.6 in the CES models and 0.8 and 1.1 in the weighted Cobb-Douglas model. This could also be due to the sources of bias having different impacts depending on the model (e.g., if capital is less accurately measured or due to the model-implied measures of productivity). We also find similar elasticity as in Hsieh and Klenow (2009) for India and China using the same model and parameterization (Panel d).

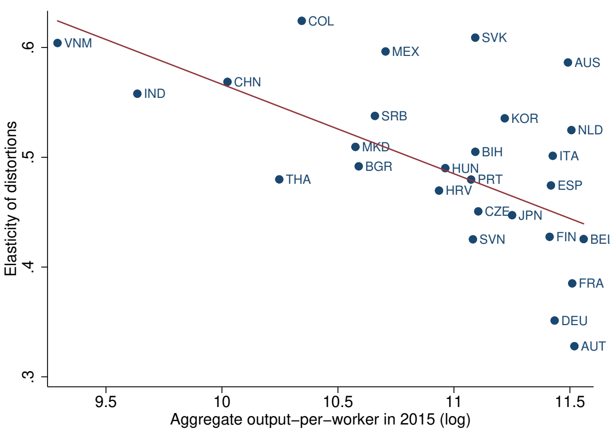
Figure B.3: Elasticity of Wedges



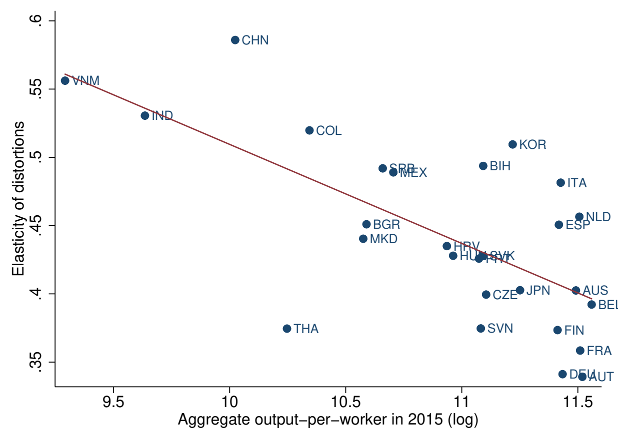
(a) Value added labor only tfp^{lo}



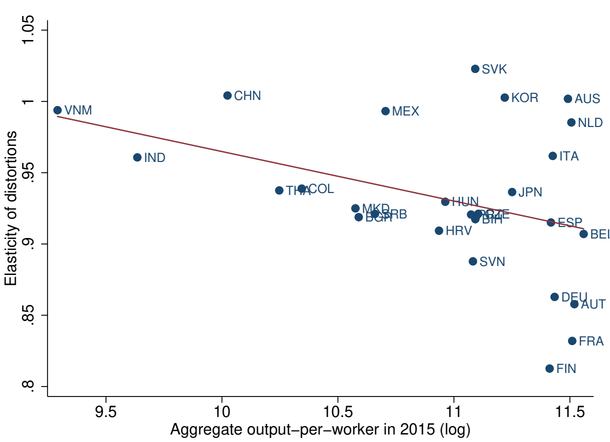
(b) Value added Cobb-Douglas tfp^{cd}



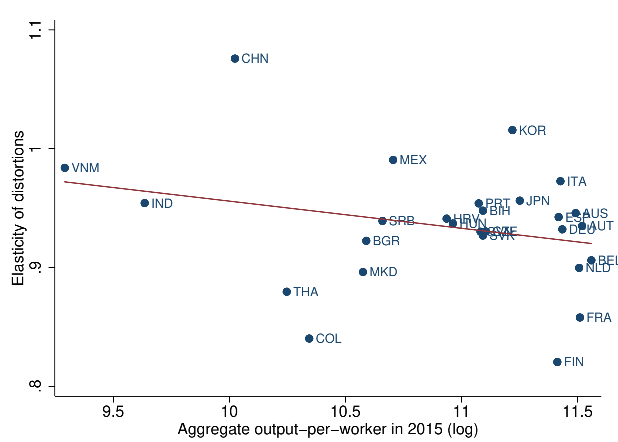
(c) CES labor only tfp^{lo}



(d) CES Cobb-Douglas tfp^{cd}



(e) Weighted labor only tfp^{lo}



(f) Weighted Cobb-Douglas tfp^{cd}

C Model Details

We provide the proofs of propositions in the paper.

Proof of Proposition 1. Simply follows from the zero-profit entry condition.

Proof of Proposition 2. Productivity can be written as

$$\begin{aligned} \ln TFP_i &= (1-\gamma)\ln z_i + \ln v_i, \\ &= \frac{1-\gamma}{\phi+\rho-1}\left[\ln \chi_i + \ln \epsilon_i\right] + \ln v_i. \end{aligned} \tag{C.1}$$

Then, the standard deviation of ln TFP across firms is given by

$$\sigma^2_{TFP} = (1-\gamma)^2\sigma^2_{z|o} + \sigma^2_v,$$

from equation (C.1) and the fact that $cov(z, v) = 0$. Define a variable $\ln \tilde{x}$ as $\ln x - \ln \bar{x}$ where $\ln \bar{x} = \mathbb{E}\ln x$. Then, going further

$$\begin{aligned} \sigma^2_{z|o} &= \mathbb{E}\left[\frac{1}{\phi+\rho-1}(\ln \chi_i + \ln \epsilon_i) - \ln \bar{z} \mid o\right]^2, \\ &= \left(\frac{1}{\phi+\rho-1}\right)^2 \mathbb{E}\left[(\ln \tilde{\chi}_i + \ln \tilde{\epsilon}_i) \mid o\right]^2, \\ &= \left(\frac{1}{\phi+\rho-1}\right)^2 \mathbb{E}\left[(\ln \tilde{\chi}_i)^2 + (\ln \tilde{\epsilon}_i)^2 + \ln \tilde{\chi}_i \ln \tilde{\epsilon}_i \mid o\right], \\ &= \left(\frac{1}{\phi+\rho-1}\right)^2 \mathbb{E}\left[\sigma^2_{\chi|o} + \sigma^2_{\epsilon|o} + cov(\chi, \epsilon|o)\right]. \end{aligned}$$

Substituting into the expression for σ^2_{TFP} confirms the result in the main text:

$$\begin{aligned} var(\text{TFP}) &= (1-\gamma)^2\sigma^2_{z|o} + \sigma^2_v, \\ &= \left(\frac{1-\gamma}{\phi+\rho-1}\right)^2 (\sigma^2_{\chi|o} + \sigma^2_{\epsilon|o} + 2\ cov(\chi, \epsilon|o)) + \sigma^2_v. \end{aligned}$$

Proof of Proposition 3. The elasticity of distortions (wedge) with respect to firm-level productivity is given by

$$elas(wedge_i, TFP_i) = \frac{\mathbb{E}[\ln \tilde{wedge}_i \ln T\tilde{F}P_i]}{\sigma^2_{TFP}}.$$

The numerator is equal to

$$\begin{aligned}
\mathbb{E}[\ln \tilde{wedge}_i \ln T\tilde{F}P_i] &= \mathbb{E}\left[((1-\gamma)\ln\tilde{z}_i + \ln\tilde{v}_i)(\ln\tilde{v}_i + \rho(1-\gamma)\ln\tilde{z}_i - (1-\gamma)\ln\tilde{\epsilon}_i)\right], \\
&= \mathbb{E}\begin{bmatrix} (1-\gamma)\ln\tilde{z}_i\ln\tilde{v}_i + \rho(1-\gamma)^2(\ln\tilde{z}_i)^2 - (1-\gamma)^2\ln\tilde{z}_i\ln\tilde{\epsilon}_i \\ +(\ln v_i)^2 + \rho(1-\gamma)\ln\tilde{z}_i\ln\tilde{v}_i - (1-\gamma)\ln\tilde{v}_i\ln\tilde{\epsilon}_i \end{bmatrix}, \\
&= \rho(1-\gamma)^2\sigma^2_{z|o} + \sigma^2_v - (1-\gamma)^2 cov(z,\epsilon|o).
\end{aligned}$$

The last line follows from $\mathbb{E}\ln\tilde{z}_i\ln v_i = 0$ and $\mathbb{E}\ln v\ln\tilde{\epsilon}_i = 0$. Along with $\sigma^2_{TFP} = (1-\gamma)^2\sigma^2_{z|o} + \sigma^2_v$, the above expression confirms the result in the main text:

$$elas(\text{TFP}, wedge) = \frac{\rho(1-\gamma)^2\sigma^2_{z|o} + \sigma^2_v - (1-\gamma)^2 cov(z,\epsilon|o)}{(1-\gamma)^2\sigma^2_{z|o} + \sigma^2_v}.$$

D Quantitative Details

We provide details of the calibration to data moments for Vietnam and further results on the decomposition between the selection and technology channels in the model.

D.1 Calibration to Vietnam

Following our calibration procedure for France, we calibrate five parameters ρ, σ_ϵ, σ_χ, σ_v, and c_f to data moments for Vietnam while keeping the remaining parameters the same as in the benchmark economy. The five parameters are jointly calibrated to match the following moments: (1) the distortion-productivity elasticity, (2) the standard deviation of log distortions, (3) the standard deviation of log employment, (4) the standard deviation of log TFP, and (5) average firm size. The calibrated parameters and targeted moments are reported in Table D.2. We note that there is no change in the dispersion of technology-choice ability σ_χ compared to the benchmark economy. We also note that while the calibrated value of c_f is somewhat different from that of the benchmark economy, the difference has no substantial aggregate implication other than average firm size. As a result in our quantitative cross-country exploration in the main text we keep this parameter constant to that of the benchmark economy.

Table D.2: Calibration to Vietnamese Economy

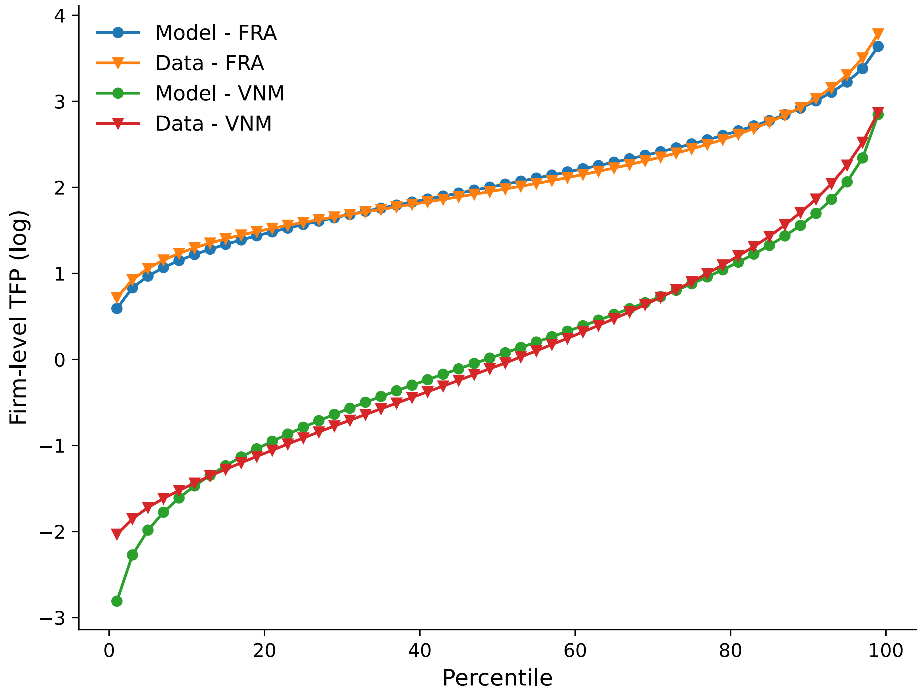
Parameter	Value	Targeted moments	Model	Data
ρ	0.90	Elasticity of distortions	0.96	0.98
σ_ϵ	3.60	sd log distortions	1.21	1.22
σ_χ	11.0	sd log employment	1.40	1.40
σ_v	0.56	sd log TFP	1.23	1.21
c_f	0.07	Average firm size	6.59	6.66

Figure D.4 illustrates the implied percentile distribution of firm-level TFP in the model compared to the French and Vietnamese data. The levels of log TFP in the French and Vietnamese data are adjusted to match the mean of log TFP implied by the calibrated models. Despite the calibration assuming a log-normal distribution of innovation abilities and targeting only the standard deviation of log TFP, the resulting distribution of firm-level TFP closely matches the Vietnamese data. The model generates lower values than the data for the bottom 5% of the TFP distribution. However, this discrepancy could be attributed to the small sample size and the trimming of the data.

D.2 Technology and Selection

Figure D.5 presents the results from experiments decomposing the effects on the standard deviation of log TFP and log employment of ex-post productivity shocks v, technology choice decision $z(\chi, \epsilon)$, and the selection channel $o(\chi, \epsilon)$. These experiments vary the value

Figure D.4: Firm-level TFP Distribution

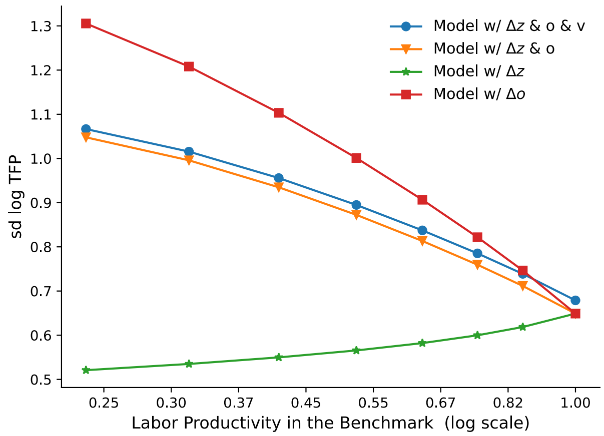


Notes: Data refers to the distribution of firm-level TFP in France and Vietnam from Orbis, whereas Model refers to the distribution of productivity in the model calibrated to France and Vietnam. To illustrate the fit of the model versus the data, the mean of the log TFP in the data is normalized to the mean in the model in each case. For ease of illustration, the figure only plots 50 percentile points of the distributions, from percentile one (p1) to percentile 99 (p99).

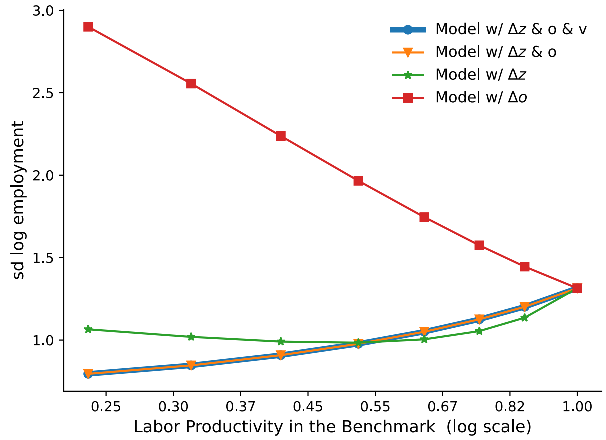
of ρ while keeping other parameters constant, including: (1) a baseline experiment with ex-post productivity shock v, changes in operating decision $z(\chi, \epsilon)$, and technology choice $z(\chi, \epsilon)$ across economies; (2) a baseline experiment but without ex-post productivity shock v; (3) an experiment without ex-post productivity shock v, maintaining operating decision $o(\chi, \epsilon)$ the same as the benchmark economy and changing only technology choice $z(\chi, \epsilon)$ across economies; and (4) an experiment without ex-post productivity shock v, maintaining technology choice $z(\chi, \epsilon)$ the same as the benchmark economy and changing only operating decision $o(\chi, \epsilon)$ changes across economies.

The ex-post productivity shock v does not generate important quantitative differences in the pattern of the standard deviation of log TFP and have no impact on the standard deviation of log employment. However, the technology and selection channels produce op-

Figure D.5: Effect of Productivity Shocks, Technology and Selection in Sample Moments



(a) Standard deviation of log TFP



(b) Standard deviation of log employment

Notes: The circle-blue line represents the sample moments in the model with productivity shock v, changes in technology choice $z(\chi, \epsilon)$, and changes in operating decision $o(\chi, \epsilon)$. The triangle-orange line reports the same but assumes no productivity shock v. The star-green and square-red lines report the same (no v) but further assume no changes in technology choice $z(\chi, \epsilon)$ and in operating decision $o(\chi, \epsilon)$ relative to the benchmark economy, respectively.

posite patterns in both the standard deviation of log TFP and log employment. While the experiment with changes in only the technology choice channel generates a positive relationship between the standard deviation of log TFP and aggregate productivity, contradicting the pattern in the data, the experiment with changes in only the selection channel produces patterns consistent with the data.

Nevertheless, changes in the selection channel create a strong negative relationship between the standard deviation of log employment and aggregate productivity, in contrast with the data pattern where there is no systematic relationship between the dispersion of log employment and aggregate productivity. The experiment with changes in technology choice effectively accounts for this dimension. These experiments highlight the important role of both technology and selection channels in generating the patterns in key micro moments consistent with the cross-country data.